3.6.09

Lin. Algebra UE

XI) 12.6: 2
12.7: 1, 2
12.8: 1, 2, 4, 5, 7α

12.6.2) $G_1 = \begin{pmatrix} 3 & 1 \\ 1 & 0 \end{pmatrix} \quad G_2 = \begin{pmatrix} 6 & 2 \\ 2 & 1 \end{pmatrix}$

ges.: $P \in GL_2(\mathbb{R})$ mit $P^T G_1 P$ und $P^T G_2 P$ Diag.-Matrizen.

$G_2 =: \iota(E,E)$, da pos. def.
$G_1 =: \sigma(E,E)$

$\sigma(E,E) = \iota(E,E) \cdot \langle E^*, f(E) \rangle$

$\Leftrightarrow \langle E^*, f(E) \rangle = (\iota(E,E))^{-1} \cdot \sigma(E,E)$

$\Leftrightarrow \quad A := G_2^{-1} \cdot G_1$

$= \frac{1}{2} \begin{pmatrix} 1 & -2 \\ -2 & 6 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$

$\chi_A(x) = \begin{vmatrix} \frac{1}{2} - x & \frac{1}{2} \\ 0 & -1-x \end{vmatrix} = (\frac{1}{2} - x)(-1-x) \quad \Rightarrow \lambda_1 = \frac{1}{2}, \; \lambda_2 = -1.$

EV zu $\lambda_1 = \frac{1}{2}$: $\quad \tilde{b}_1 := \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

EV zu $\lambda_2 = -1$: $\quad \left(\begin{array}{cc|c} \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \end{array}\right) \Rightarrow \tilde{b}_2 := \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\Rightarrow P := \begin{pmatrix} 1 & 1 \\ 0 & -3 \end{pmatrix} \in GL_2(\mathbb{R}) =: \langle E^*, \tilde{B} \rangle$

$\| \tilde{b}_1 \| = \sqrt{(1,0) \begin{pmatrix} 6 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = \sqrt{6}$

$\| \tilde{b}_2 \| = \quad \dots = \sqrt{3}$

$\Rightarrow Q := \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & \sqrt{2} \\ 0 & -3\sqrt{2} \end{pmatrix} \in O_2 \quad =: \langle E^*, B \rangle$

$\Rightarrow \langle B^*, f(B) \rangle = \operatorname{diag}(\lambda_1, \lambda_2)$

$\iota(B,B) = Q^T G_2 Q = E_2$

$\sigma(B,B) = Q^T G_1 Q \overset{E_2}{=} \operatorname{diag}(\lambda_1, \lambda_2) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$

12.7.1) $(\mathbb{R}^{3\times1}, \iota)$

$$f := \langle E^*, f(E)\rangle = \frac{1}{25}\begin{pmatrix} 9 & 12 & -40 \\ 0 & 0 & 0 \\ -12 & -16 & -30 \end{pmatrix}$$

ges.: ONB (b_1, b_2, b_3) von $\mathbb{R}^{3\times1}$ mit $(f(b_1), f(b_2))$ OGS von $\mathbb{R}^{3\times1}$, Ker $f = [b_3]$
Sing.-Werte von f.

$$\hat{f}\circ f = \langle E^*, f(E)\rangle^T \langle E^*, f(E)\rangle = \frac{1}{25^2}\begin{pmatrix} 9 & 0 & -12 \\ 12 & 0 & -16 \\ -40 & 0 & -30 \end{pmatrix}\begin{pmatrix} 9 & 12 & -40 \\ 0 & 0 & 0 \\ -12 & -16 & -30 \end{pmatrix} = \frac{1}{25}\begin{pmatrix} 9 & 12 & 0 \\ 12 & 16 & 0 \\ 0 & 0 & 100 \end{pmatrix}$$

$$\chi_{\hat{f}\circ f}(x) = x^3 - 5x^2 + 4x = x(x-1)(x-4) \Rightarrow \lambda_1 = 0;\ \lambda_2 = 1;\ \lambda_3 = 4 \text{ ... EW v. } \hat{f}\circ f$$

$$\Rightarrow \cancel{s_1 = 0};\ s_2 = 1;\ s_3 = 2 \text{ ... SW v. } f.$$

EV zu $\lambda_3 = 4$: $\begin{pmatrix} -91 & 12 & 0 \\ 12 & -84 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\cdot 7} \begin{pmatrix} -91 & 12 & 0 \\ 625 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \tilde{b}_1 := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

EV zu $\lambda_2 = 1$: $\begin{pmatrix} -16 & 12 & 0 \\ 12 & -9 & 0 \\ 0 & 0 & 3 \end{pmatrix} \xrightarrow{\cdot \frac{4}{3}} \begin{pmatrix} 0 & 0 & 0 \\ 4 & -3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \tilde{b}_2 := \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

EV zu $\lambda_1 = 0$: $\begin{pmatrix} 9 & 12 & 0 \\ 12 & 16 & 0 \\ 0 & 0 & 4 \end{pmatrix} \xrightarrow{\cdot -\frac{4}{3}} \begin{pmatrix} 3 & 4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow \tilde{b}_3 := \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix}$

$\|\tilde{b}_1\| = 1 \Rightarrow b_1 := \tilde{b}_1 = (0, 0, 1)^T$

$\|\tilde{b}_2\| = 5 \Rightarrow b_2 := \frac{1}{5}\tilde{b}_2 = (\frac{3}{5}, \frac{4}{5}, 0)^T$

$\|\tilde{b}_3\| = 5 \Rightarrow b_3 := \frac{1}{5}\tilde{b}_3 = (-\frac{4}{5}, \frac{3}{5}, 0)^T$

12.7.2) $f_A \in L(\mathbb{R}^{3\times1}, \mathbb{R}^{4\times1})$

$$A := \begin{pmatrix} 2 & 0 & -10 \\ -11 & 0 & 5 \\ 0 & 3 & 0 \\ 0 & -4 & 0 \end{pmatrix}$$

ges.: ONB $B := (b_1, b_2, b_3)$ von $\mathbb{R}^{3\times1}$ mit $f(B)$ OGS von $\mathbb{R}^{4\times1}$. SW von f.

$$A^T\cdot A = \begin{pmatrix} 2 & -11 & 0 & 0 \\ 0 & 0 & 3 & -4 \\ -10 & 5 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & -10 \\ -11 & 0 & 5 \\ 0 & 3 & 0 \\ 0 & -4 & 0 \end{pmatrix} = \begin{pmatrix} 125 & 0 & -75 \\ 0 & 25 & 0 \\ -75 & 0 & 125 \end{pmatrix}$$

$$\chi_{A^T\cdot A}(x) = -x^3 + 275x^2 \mp 16.250x + 25000 = (25-x)(50-x)(200-x)$$

$$\Rightarrow \lambda_1 = 25,\ \lambda_2 = 50,\ \lambda_3 = 200 \text{ ... EW v. } A^T\cdot A = \hat{f}_A\circ f_A$$

$$\Rightarrow \sigma_1 = 5,\ \sigma_2 = 5\sqrt{2},\ \sigma_2 = 10\sqrt{2} \text{ ... SW v. } f.$$

EV zu $\lambda_1 = 25$: $\begin{pmatrix} 100 & 0 & -75 \\ 0 & \cancel{25}\,0 & 0 \\ -75 & 0 & 100 \end{pmatrix} \Rightarrow \tilde{b}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

EV zu $\lambda_2 = 50$: $\begin{pmatrix} 75 & 0 & -75 \\ 0 & -25 & 0 \\ -75 & 0 & 75 \end{pmatrix} \Rightarrow \tilde{b}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

EV zu $\lambda_3 = 200$: $\begin{pmatrix} -75 & 0 & -75 \\ 0 & -175 & 0 \\ -75 & 0 & -75 \end{pmatrix} \Rightarrow \tilde{b}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

$\Rightarrow B = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}$

12.8.1) $f := A = \begin{pmatrix} -6 & -4 \\ 13 & 6 \end{pmatrix} \in GL_2(\mathbb{R})$

ges.: Polarzerlegung $A = QS$ mit $Q \in O_2$, $S \in \mathbb{R}^{2\times 2}$ pos. semidef.

$\hat{f} \circ f = A^T \cdot A = \begin{pmatrix} -6 & 13 \\ -4 & 6 \end{pmatrix} \begin{pmatrix} -6 & -4 \\ 13 & 6 \end{pmatrix} = \begin{pmatrix} 205 & 102 \\ 102 & 52 \end{pmatrix}$

$\Rightarrow \chi_{\hat{f}\circ f} = X^2 - 257X + 256 = (X-1)(X-256) \Rightarrow \lambda_1 = 1;\ \lambda_2 = 256$

EV zu $\lambda_1 = 1$: $\begin{pmatrix} 204 & 102 \\ 102 & 51 \end{pmatrix} \Rightarrow b_1 := \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

EV zu $\lambda_2 = 256$: $\begin{pmatrix} -51 & 102 \\ 102 & -204 \end{pmatrix} \Rightarrow b_2 := \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\Rightarrow \langle E^*, B \rangle = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

$\langle B^*, (\hat{f}\circ f)(B) \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 256 \end{pmatrix} \Rightarrow \langle B^*, \sqrt{\hat{f}\circ f}(B) \rangle = \begin{pmatrix} 1 & 0 \\ 0 & 16 \end{pmatrix}$

$S := \langle E^*, \sqrt{\hat{f}\circ f}(E) \rangle = \langle E^*, B \rangle \langle B^*, \sqrt{\hat{f}\circ f}(B) \rangle (\langle E^*, B \rangle)^{-1}$

$= \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 16 \end{pmatrix} \frac{1}{5} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 13 & 6 \\ 6 & 4 \end{pmatrix}$

$A = QS \Rightarrow A \cdot S^{-1} = Q$

$A \cdot S^{-1} = \begin{pmatrix} -6 & -4 \\ 13 & 6 \end{pmatrix} \frac{1}{16} \begin{pmatrix} 4 & -6 \\ -6 & 13 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} =: Q$

$A = QS \Leftrightarrow A^T = (QS)^T = S^T \cdot Q^T$

Suche Polarzerlegung für $\tilde{A} := A^T = \begin{pmatrix} -6 & 13 \\ -4 & 6 \end{pmatrix}$, $\tilde{A} = \tilde{Q}\tilde{S}$

$$A^{TT} \cdot A^T = A \cdot A^T = \begin{pmatrix} 52 & -102 \\ -102 & 205 \end{pmatrix}$$

$$\chi_{f\circ\hat{f}}(x) = \chi_{\hat{f}\circ f}(x)$$

$$\tilde{\lambda}_1 = \lambda_2, \; \tilde{\lambda}_2 = \lambda_1 \Rightarrow \langle E^*, \tilde{B} \rangle = \langle E^*, B \rangle$$

$$\langle B^*, \sqrt{\hat{f}\circ f}(B) \rangle = \begin{pmatrix} 16 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \tilde{S} := \begin{pmatrix} 4 & -6 \\ -6 & 13 \end{pmatrix}, \quad \tilde{Q} := \tilde{A} \cdot \tilde{S}^{-1} = Q^T$$

$$\tilde{A} = \tilde{Q}\tilde{S} \Leftrightarrow \tilde{A}^T = A^{TT} = A = \cancel{Q}\, \tilde{S}^T \cdot \underbrace{\tilde{Q}^T}_{Q} = \begin{pmatrix} 4 & -6 \\ -6 & 13 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

12.8.2) $A = \begin{pmatrix} 7 & 14 & -\frac{77}{5} \\ 0 & -5 & 9 \\ 0 & -16 & 19 \end{pmatrix}$

ges.: $P \in O_3$ mit $P^{-1} \cdot A \cdot P$ ist obere Dreiecksmatrix

$$\begin{aligned} \chi_A(x) &= (7-x)(-5-x)(19-x) + 16 \cdot 9\,(7-x) \\ &= (7-x)\big((-5-x)(19-x) + 16 \cdot 9\big) \\ &= (7-x)(x^2 + 14x + 49) \\ &= (7-x)^3 \end{aligned}$$

$\Rightarrow \lambda = 7$, VF: 3

$$\left(\begin{array}{ccc|ccc} 0 & 14 & -\frac{77}{5} & 1 & 0 & 0 \\ 0 & -12 & 9 & 0 & 1 & 0 \\ 0 & -16 & 12 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \cdot \frac{7}{6} \\ \cdot \left(-\frac{4}{3}\right) \end{array}$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & -\frac{49}{10} & 1 & \frac{7}{6} & 0 \\ 0 & -12 & 9 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\frac{4}{3} & 1 \end{array}\right)$$

$\vdots$

$$\left(\begin{array}{ccc|ccc} 0 & 1 & 0 & -\frac{15}{98} & -\frac{11}{42} & 0 \\ 0 & 0 & 1 & -\frac{10}{49} & -\frac{5}{21} & 0 \\ 0 & 0 & 0 & 0 & -\frac{4}{3} & 1 \end{array}\right)$$

also:

$$\left(\begin{array}{ccc|c} 0 & 14 & -\frac{77}{5} & 0 \\ 0 & -12 & 9 & 0 \\ 0 & -16 & 12 & 0 \end{array}\right) \Rightarrow a_{11} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 0 & 14 & -\frac{77}{5} & 1 \\ 0 & -12 & 9 & 0 \\ 0 & -16 & 12 & 0 \end{array}\right) \Leftrightarrow \left(\begin{array}{ccc|c} 0 & 1 & 0 & -\frac{15}{98} \\ 0 & 0 & 1 & -\frac{10}{49} \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow a_{12} = \begin{pmatrix} 0 \\ -\frac{15}{98} \\ -\frac{10}{49} \end{pmatrix} = \frac{5}{98} \begin{pmatrix} 0 \\ -3 \\ -4 \end{pmatrix}$$

$$\left(\begin{array}{c|c} \text{—"—} & \begin{matrix} 0 \\ -\frac{15}{98} \\ -\frac{10}{49} \end{matrix} \end{array}\right) \Leftrightarrow \left(\begin{array}{c|c} \text{—"—} & \begin{matrix} \frac{55}{1372} \\ \frac{25}{686} \\ 0 \end{matrix} \end{array}\right) \Rightarrow a_{13} = \begin{pmatrix} 0 \\ \frac{55}{1372} \\ \frac{25}{686} \end{pmatrix} = \frac{5}{1372} \begin{pmatrix} 0 \\ 11 \\ 10 \end{pmatrix}$$

$\cancel{\Rightarrow \tilde{P} = (a_{11}, a_{12}, a_{13}) \ldots \text{OGS}}$

ges.: ONB mit $[a_{11}, a_{12}, a_{13}] = [b_1, b_2, b_3]$

·) Schmidt: $\{a_{11}, \tilde{a}_{12}\}$ bilden OGS, $\tilde{a}_{12} := (0, 3, 4)^T$

$$\tilde{a}_{13} = \begin{pmatrix} 0 \\ 11 \\ 10 \end{pmatrix} - \frac{73}{25} \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = \frac{14}{25} \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix} \Rightarrow \tilde{a}_{13} := (0, 4, -3)^T$$

·) normalisieren:

$$\Rightarrow P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{5} & \frac{4}{5} \\ 0 & \frac{4}{5} & -\frac{3}{5} \end{pmatrix} \in O_3$$

Probe: $P^{-1} \cdot A \cdot P = \begin{pmatrix} 7 & -\frac{83}{25} & \frac{531}{25} \\ 0 & 7 & -25 \\ 0 & 0 & 7 \end{pmatrix}$

12.8.4) $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \in \mathbb{R}^{3\times 3}$

ges.: zu A orth. ähnliche Hessenberg-Matrix.

$c_1 := e_1, \; b_2 := e_2, \; b_3 := e_3$

<u>1. Schritt:</u> Ortho.-Proj. v. $f(c_1)$ auf $[b_2, b_3]$

$f(c_1) = \cancel{e_1} A \cdot e_1 = (0,0,1)^T = e_3$

$\Rightarrow \underset{\text{O-Proj.}}{pr}(f(c_1)) = pr(e_3) = e_3 =: c_1'$

<u>2. Schritt:</u> c_1', b_2 l.u. $\Rightarrow$ Basistransformation durch ortho. Spiegelung

$b_2' := \|c_1'\| \, b_2 = b_2 = e_2$

$\Rightarrow s_+ := \frac{1}{\|b_2' + c_1'\|} (b_2' + c_1') = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$\Rightarrow P := E_3 - \cancel{2}\frac{1}{\cancel{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} (0,1,1)$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \in O_3$

$P^T A P = \begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ \boxed{0} & 0 & 1 \end{pmatrix}$ ... Hessenberg-Matrix.

12.8.7) $A = \frac{1}{5} \begin{pmatrix} -3 & 3 & 7 \\ -4 & 4 & 1 \\ 0 & 5 & 0 \end{pmatrix}$

ges.: $Q \in O_3$, R obere Dreiecksmatrix mit $A = QR$.

<u>$k=0$:</u> $A = Q_0 R_0 = E_3 A$

<u>1. Schritt:</u> Ortho.-Proj. v. $r_1 := (-\frac{3}{5}, -\frac{4}{5}, 0)$ auf $[e_1, e_2, e_3]$

$\Rightarrow r_1' := r_1$

<u>2. Schritt:</u> r_1', e_1 l.u. $\Rightarrow$ ortho. Spiegelung

$e_1' := \frac{-3}{|-3|} \|r_1'\| e_1 = -e_1$

$\Rightarrow s_+ := \frac{1}{\|e_1' + r_1'\|} (e_1' + r_1') = \frac{5}{4\sqrt{5}} \begin{pmatrix} -\frac{8}{5} \\ -\frac{4}{5} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix}$

$\Rightarrow S_0 := E_3 - 2\frac{1}{5} \begin{pmatrix} -2 \\ -1 \\ 0 \end{pmatrix} (-2,-1,0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{2}{5} \begin{pmatrix} 4 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -3 & -4 & 0 \\ -4 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

3. Schritt:

$$Q_1 := Q_0 S_0 = E_3 S_0 = S_0 \in O_3$$

$$R_1 := S_0 R_0 = S_0 A = \frac{1}{25}\begin{pmatrix}-3&-4&0\\-4&3&0\\0&0&5\end{pmatrix}\begin{pmatrix}-3&3&7\\-4&4&1\\0&5&0\end{pmatrix} = \begin{array}{c}\begin{array}{ccc}\overset{T_1}{\downarrow}&&\end{array}\\ \begin{pmatrix}\boxed{1}&-1&-1\\0&0&-1\\0&1&0\end{pmatrix}\end{array}$$

$k=1$: erkennen: R_1 kann durch Zeilenvertauschung auf obere Dreiecksmatrix umgeformt werden.

$$R_1 = \begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}\underbrace{\begin{pmatrix}1&-1&-1\\0&1&0\\0&0&1\end{pmatrix}}_{=:R}$$

$$\Rightarrow A = Q_1 R_1 = \underbrace{Q_1\begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix}}_{=:Q \in O_3} R \quad \Rightarrow Q := \frac{1}{5}\begin{pmatrix}-3&0&4\\-4&0&-3\\0&5&0\end{pmatrix}$$

ges.: 1. Schritt des QR-Verfahrens (Berechnung von A_1)

$$A =: A_0 = QR$$

$$A_1 := Q^T A_0 Q = \underbrace{Q^T Q}_{E_3} R Q = RQ$$

$$\Rightarrow A_1 = \frac{1}{5}\begin{pmatrix}1&-5&7\\-4&0&-3\\0&5&0\end{pmatrix}$$

Alternative mit Anwendung von A 12.8.5:

Schmidtsches Orthonormalisierungsverfahren:

$$q_1 := a_1 = \frac{1}{5}\begin{pmatrix}-3\\-4\\0\end{pmatrix}$$

$$q_2 := a_2 + a_1 = \frac{1}{5}\begin{pmatrix}0\\0\\5\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$q_3 := b_3 - (a_1 \cdot b_3)\, a_1 = \frac{1}{5}\begin{pmatrix}7\\1\\0\end{pmatrix} + \frac{1}{5}\begin{pmatrix}-3\\-4\\0\end{pmatrix} = \frac{1}{5}\begin{pmatrix}4\\-3\\0\end{pmatrix}$$

$$\Rightarrow Q := \frac{1}{5}\begin{pmatrix}-3&0&4\\-4&0&-3\\0&5&0\end{pmatrix}, \quad R := Q^{-1} A.$$