

Aufgabe 1

$$\langle E^+, f(E) \rangle = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ -1 & 0 & 3 & 1 \\ 0 & 1 & 0 & 3 \end{pmatrix}$$

a) ges.: $\chi_f(x)$, EW von f

$$\chi_f(x) = \det(f - x \cdot \text{id}_V) = \begin{vmatrix} 3-x & 1 & 0 & 0 \\ 0 & 3-x & 0 & 0 \\ -1 & 0 & 3-x & 1 \\ 0 & 1 & 0 & 3-x \end{vmatrix}$$

$$= (3-x) \begin{vmatrix} 3-x & 1 & 0 \\ 0 & 3-x & 0 \\ 0 & 1 & 3-x \end{vmatrix}$$

$$= (3-x)^2 \begin{vmatrix} 3-x & 1 \\ 0 & 3-x \end{vmatrix} = (3-x)^4$$

\Rightarrow EW $\lambda = 3$, \forall unth. VF: 4

b) ges.: zu $\langle E^+, f(E) \rangle$ char. Matrix in JNF

\overline{B}_3 $\langle B^+, f(B) \rangle$ in JNF

Eigenraum zu $\lambda = 3$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 \end{array}$$

$$\Rightarrow \mathcal{B}_{11} := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathcal{B}_{21} := \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Beispielraum

$$\begin{array}{cccc|c|c} x_1 & x_2 & x_3 & x_4 & & \\ \hline 0 & 1 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$\Rightarrow \mathcal{B}_{12} := \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathcal{B}_{22} := \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \langle \mathbb{B}^*, f(\mathbb{B}) \rangle = \left(\begin{array}{|c|c|} \hline 3 & 1 \\ \hline & 3 \\ \hline \end{array} \right)$$

$$B = \langle E^*, B \rangle = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$d) \mu_f(x) = (3-x)^2$$

Aufgabe 2

$$\langle E^*, f_0(E) \rangle = \begin{pmatrix} 3 & 3 & -3 & 0 \\ -3 & 0 & 0 & 0 \\ 3 & 3 & -3 & 0 \\ -3 & 0 & 0 & 0 \end{pmatrix}$$

a) ges.: $\chi_{f_0}(x)$, EW v. f_0

$$\chi_{f_0}(x) = \det(f_0 - x \cdot \text{id}_V) = \begin{vmatrix} 3-x & 3 & -3 & 0 \\ -3 & -x & 0 & 0 \\ 3 & 3 & -3-x & 0 \\ -3 & 0 & 0 & -x \end{vmatrix}$$

$$= -x \begin{vmatrix} 3-x & 3 & -3 \\ -3 & -x & 0 \\ 3 & 3 & -3-x \end{vmatrix}$$

$$= -x \left((3-x) \cdot (3+x) + 27 - 9x - 9(3-x) \right)$$

~~$= (-x)(3+x)$~~

$$= -x \left((3-x) \cdot (3+x) \right)$$

$$= -x \left((3-x) \cdot (3+x) + 27 - 9x - 9(3+x) \right)$$

$$= -x \left((3-x) \cdot (3+x) - 18x \right)$$

$$= -x^2 (9 - x^2 - 18)$$

$$= +x^2 (x^2 + 9)$$

$$\Rightarrow \text{EW: } \chi_{f_0}(x) = x^2(x^2 + 9) = 0$$

$$\Leftrightarrow x^2 = 0 \Leftrightarrow \lambda_{1,2} = 0$$

$$\Leftrightarrow x^2 + 9 = 0 \Leftrightarrow \lambda_3 = 3i \\ \lambda_4 = -3i$$

b) ges.: C - Basis v. $\mathbb{C}^{4 \times 4}$ mit $\langle e^*, f_C(C) \rangle$ ist Diag.

EV zu $\lambda_1, \lambda_2 = 0$

$$\begin{array}{cccc|c}
 x_1 & x_2 & x_3 & x_4 & \\
 \hline
 3 & 3 & -3 & 0 & 0 \\
 -3 & 0 & 0 & 0 & 0 \\
 3 & 3 & -3 & 0 & 0 \\
 3 & 0 & 0 & 0 & 0 \\
 \hline
 1 & 1 & -1 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 \\
 & & 0 & 0 & 0
 \end{array}$$

$$\Rightarrow e_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

~~EV zu λ~~

EV zu $\lambda_3 = 3i$: es. Hinweis: $e_3 := \begin{pmatrix} i \\ -1 \\ i \\ -1 \end{pmatrix}$

\Rightarrow EV zu $\lambda_4 = -3i$: $e_4 := \overline{e_3} = \begin{pmatrix} i \\ 1 \\ i \\ 1 \end{pmatrix}$

$$C = \begin{pmatrix} 0 & 0 & i & i \\ 0 & 1 & -1 & 1 \\ 0 & 1 & i & i \\ 1 & 0 & -1 & 1 \end{pmatrix} ; \quad \langle e^*, f_C(C) \rangle = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 3i & \\ & & & -3i \end{pmatrix}$$

c) $\mathcal{A} \langle B^*, f(B) \rangle$ in reellen JWF

$$\langle B^*, f(B) \rangle = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & -3 \\ & & 3 & 0 \end{pmatrix}$$

d) $B = \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & -2 \end{pmatrix}$!

Aufgabe 3

$$G = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}$$

a) ges: $P \in GL_3(\mathbb{C})$ mit $A := P^T G P$, wobei A reals. Matrix in Normalform.

$$\begin{array}{ccc} \begin{array}{ccc} & \cdot i & \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & -i & 0 \\ -i & i & 0 & i \\ 0 & -i & 0 \end{array} & \xrightarrow{\mathbb{Z} \rightarrow \mathbb{S}} & \begin{array}{ccc} & \cdot -1 & \\ 1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 1 \\ \hline 0 & -1 & 0 \\ +1 & 0 & +1 \\ 0 & -1 & 0 \end{array} \xrightarrow{\mathbb{Z} \rightarrow \mathbb{S}} \begin{array}{ccc} 1 & 0 & -1 \\ 0 & -i & 0 \\ 0 & 0 & 1 \\ \hline 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} \end{array}$$

$$\Rightarrow P = \begin{pmatrix} 1 & 0 & -1 \\ 0 & -i & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

b) ges: $Q \in GL_3(\mathbb{C})$ mit $H := \bar{Q}^T G Q$, wobei H Herm. Matrix in Normalform.

$$\begin{array}{ccc} \begin{array}{ccc} & \cdot i & \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 0 & -i & 0 \\ -i & i & 0 & i \\ 0 & -i & 0 \end{array} & \xrightarrow{\mathbb{Z} \rightarrow \mathbb{S}} & \begin{array}{ccc} & & \\ 1 & 0 & 0 \\ 0 & +i & 0 \\ 0 & 0 & 1 \\ \hline 0 & 1 & 0 \\ +1 & 0 & 1 \\ 0 & 1 & 0 \end{array} \xrightarrow{\mathbb{S}} \begin{array}{ccc} 1 & 0 & 0 \\ i & i & 0 \\ 0 & 0 & 1 \\ \hline 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array} \end{array}$$

$$\begin{array}{ccc} \begin{array}{ccc} \times & & \\ \hline 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array} & \xrightarrow{\mathbb{S}} & \begin{array}{ccc} 1 & -\frac{1}{2} & -\frac{1}{2} \\ \hline 2 & 0 & 0 \\ 1 & -\frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{array} \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 \times \\
 \hline
 2 \ 0 \ 0 \\
 -2 \ 0 \ -\frac{1}{2} \ \frac{1}{2} \\
 -2 \ 0 \ \frac{1}{2} \ -\frac{1}{2} \\
 \cdot 2 \cdot 2
 \end{array}
 \xrightarrow{s} \begin{array}{c}
 \begin{array}{ccc}
 1 & -1 & -1 \\
 i & i & -i \\
 0 & 0 & 1
 \end{array} \\
 \hline
 \begin{array}{ccc}
 2 & 0 & 0 \\
 0 & -2 & 2 \\
 0 & 2 & -2
 \end{array}
 \end{array}
 \xrightarrow{z} \begin{array}{c}
 \begin{array}{ccc}
 1 & -1 & -2 \\
 i & i & 0 \\
 0 & 0 & 2
 \end{array} \\
 \hline
 \begin{array}{ccc}
 2 & 0 & 0 \\
 0 & -2 & 0 \\
 0 & 0 & 0
 \end{array}
 \end{array}
 \cdot \frac{1}{\sqrt{2}}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 \textcircled{Q} \\
 \hline
 1 \ 0 \ 0 \\
 0 \ -1 \ 0 \\
 0 \ 0 \ 0
 \end{array}
 \} = H
 \end{array}$$

$$Q = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -2 \\ i & i & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i & -2\sqrt{2} \\ i & 1 & 0 \\ 0 & 0 & 2\sqrt{2} \end{pmatrix}$$

$$-\frac{1}{i} = \frac{i \cdot i}{i} = \frac{-1}{i} = \frac{1}{\sqrt{2}}$$

$$(-2) \left(-\frac{1}{\sqrt{2}}\right) \frac{1}{\sqrt{2}} = -1$$

$$\frac{1}{\sqrt{-2}} = \frac{1}{\sqrt{2}i} = \frac{1}{\sqrt{2}i} \cdot \frac{i}{i} = \frac{i}{-2} = -\frac{i}{2}$$

$$(-2) \left(-\frac{1}{\sqrt{2}i}\right)$$

$$= 2 \frac{1}{2i^2} = -1$$

$$\cdot (x_1, x_2, x_3) \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix} = (i x_2, -i(x_1+x_3), i x_2) = (0, 0, 0)$$

c)

$$\Leftrightarrow x_2 = 0$$

$$x_1 + x_3 = 0$$

\Rightarrow Basis:

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

d) indefinit

Aufgabe 4

$$\sigma_1(E, E) \approx \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

$$\sigma_2(E, E) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

a) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & -1 & 0 \end{pmatrix} \xrightarrow{\substack{I_1 - I_2 \\ I_3 - I_2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \xrightarrow{I_3 - I_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix} \xrightarrow{I_3 - I_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{I_2 \cdot (-1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{\substack{I_2 \leftrightarrow I_3 \\ I_2 \cdot \frac{1}{2}}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{I_3 - I_2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{I_2 \cdot (-1)} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

b) $(+, -, 0)$
 $(+, +, 0)$

c) n. Rangwert, die versch. Signaturen

$$d) (\sigma_1)_c \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{I_2 \cdot (-1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(\sigma_2)_c \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

e) ja, die Rang. zu selber Matrix im Normalform
 (do selber Rang)

$\xrightarrow{S} \xrightarrow{Z}$

$$\begin{array}{ccc|ccc} 1 & -1 & -2 & & & \\ -i & -i & 2 & & & \\ \hline 0 & 0 & 2 & & & \\ 2 & 0 & 0 & & & \\ 0 & -2 & 0 & & & \\ 0 & 0 & 0 & & & \end{array} \xrightarrow{S} \xrightarrow{Z}$$

$$P \begin{array}{ccc|ccc} \hline 1 & 0 & 0 & & & \\ 0 & -1 & 0 & & & \\ 0 & 0 & 0 & & & \\ \hline \end{array} \} =: G$$

$$\frac{1}{\sqrt{4i}} = \frac{1}{2\sqrt{i}} = \frac{1}{2 \cdot \sqrt{1+i}} = \frac{1}{2\sqrt{1+i}}$$

$$= \frac{1}{2(-i)} = \frac{i}{2}$$

$$P := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & -2\sqrt{2} \\ -i & -i & 2\sqrt{2} \\ 0 & 0 & 2\sqrt{2} \end{pmatrix}$$

b) indednet

c) nein, die nicht vertikal

$$d) (3-2i, -i, -1) \begin{pmatrix} 0 & i & -i \\ -i & 0 & -i \\ i & i & 0 \end{pmatrix} \cdot \begin{pmatrix} -1-i \\ 2i+2 \\ -3i-3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= -(1+i)x_1 + 2(i+1)x_2 - 3(i+1)x_3 = (i+1)(x_1 - 2x_2 + 3x_3) = 0$$

$$\Leftrightarrow x_1 - 2x_2 + 3x_3 = 0$$

Aufgabe 6

$$G = \begin{pmatrix} 1 & i & 0 \\ -i & 2 & -i \\ 0 & i & 2 \end{pmatrix}$$

a) $G_{(1)} = 1$

$$G_{(2)} = \begin{vmatrix} 1 & i \\ -i & 2 \end{vmatrix} = 1$$

$$G_{(3)} = \begin{vmatrix} 1 & i & 0 \\ -i & 2 & -i \\ 0 & i & 2 \end{vmatrix} = 4 - 1 - 2 = 1$$

b) $\text{rg } G = 3$, da $\det G \neq 0$

c) G pos. def., da alle Hauptminoren > 0

d)

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & i & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \xrightarrow{S} \begin{array}{ccc|ccc} 1 & i & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -i \\ 0 & 0 & 1 & 0 & i & 2 \end{array} \xrightarrow{Z} \begin{array}{ccc|ccc} \times & & & 1 & 0 & 0 \\ & & & 0 & 1 & -i \\ & & & 0 & i & 2 \end{array}$$

$$\begin{array}{ccc|ccc} 1 & -i & 1 & 1 & 0 & 0 \\ 0 & 1 & i & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \xrightarrow{S} \begin{array}{ccc|ccc} 1 & -i & 1 & 1 & 0 & 0 \\ 0 & 1 & i & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \xrightarrow{Z} \begin{array}{ccc|ccc} 1 & -i & \frac{1}{\sqrt{2}} & 1 & 0 & 0 \\ 0 & 1 & i\sqrt{2} & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 1 \end{array}$$

~~... =: Q~~

$$\begin{aligned} E_n &= \overline{Q}^T \cdot G \cdot Q \Leftrightarrow G = (\overline{Q}^T)^{-1} \cdot E_n \cdot Q^{-1} \\ &= (\overline{Q^{-1}})^T \cdot E_n \cdot Q^{-1} =: P \\ \Rightarrow G &= P^T \cdot E_n \cdot P \end{aligned}$$

$$\begin{array}{ccc} 1 & -i & \cancel{1} \\ 0 & 1 & \cancel{i} \\ 0 & 0 & \cancel{1} \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$



→

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & i \\ 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc} 1 & i & -1 \\ 0 & 1 & 0 \\ 0 & 0 & \cancel{1} \end{array}$$

→

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc} 1 & i & 0 \\ 0 & 1 & \cancel{-i} \\ 0 & 0 & \cancel{1} \end{array} \} =: P$$