Occupational safety in a frictional labor market

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Abstract

This paper studies the provision of occupational safety when the labor market is subject to search frictions. While safety measures are costly for firms, they reduce workers' mortality. We show that the presence of search frictions decreases the socially optimal level of occupational safety relative to a frictionless labor market, leading to excess mortality. In a decentralized setting where wages and safety measures are bargained at the firm level, matching externalities and a labor supply externality may further reduce safety provision. We obtain conditions under which these externalities are internalized by firms and workers, and discuss the role of policy for promoting occupational safety.

Keywords: occupational safety, mortality, search frictions, Nash bargaining

JEL classification: J17, J28, J32, J38, J64

1 Introduction

According to the European Agency for Safety and Health at Work (2017a,b), an annual number of 2.8 million deaths worldwide can be attributed to work-related injuries and diseases, amounting to 67.8 million years of life lost. Additionally, non-fatal work-related injuries and diseases cause 55.5 million years lived in disability. Valued by the average production of a worker, the estimated economic costs of fatal and non-fatal incidents equal 3.9% of global GDP. These costs are also sizeable in high income countries, where cancer, musculoskeletal disorders, and circulatory diseases are the most prevalent work-related health issues.[1]

Consequently, safety and health at the workplace have been identified as key for prolonging working lives and healthy aging, resulting in broad policy initiatives like those of the European Commission (2021a,b) as well as actions specifically targeted at protecting workers from COVID-19 (Biden, 2021). From a normative perspective, policy intervention in occupational safety

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[1] In the EU-28, the costs are estimated at 3.3% of GDP (European Agency for Safety and Health at Work, 2017a). Country-specific studies that use more granular cost estimation models report GDP shares of 1% for the UK, 1.8% for the US, 2.9% for Finland, 3.2% for Singapore, 3.5% for Germany and the Netherlands, 4.8% for Australia, 6.3% for Italy, and 10.2% for Poland, see Tompa et al. (2021) and references therein. The large range of estimates is partly due to different cost categories considered.

provision[2] can be socially desirable since the level of safety measures arising from the interplay of firm and worker incentives is likely to be inefficient (Henderson, 1983). This is due to the presence of asymmetric information about health risks, psychological biases in the individual perception of risk, as well as externalities on co-workers and society that individual firms and workers do not take into account (Pouliakas and Theodossiou, 2013).

Another source of inefficiencies that so far has received little attention in the context of occupational safety, are labor market frictions in the matching of unemployed to job openings. Stronger frictions increase the time that unemployed need to find and take up a job. The less frequent they get the opportunity to work, the higher will be their willingness to accept jobs with low safety standards. On the other hand, frictions also increase the time that firms need to fill a vacancy. The longer it takes them to find an applicant for an open position, the higher is their incentive to safeguard worker health once a match is formed. Due to these opposing effects, the impact of search frictions on occupational safety is a priori not clear.

This paper studies the provision of occupational safety in the presence of search frictions as featured in the workhorse model of modern labor economics, the Diamond-Mortensen-Pissarides (DMP) model. Since occupational safety ultimately affects workers' mortality, we extend the basic DMP model (Pissarides, 2000, Ch. 1) for mortality shocks. The mortality rate of employed individuals is endogenously determined and our main variable of interest. We solve three versions of our model to identify (i) the mortality effect of search frictions and (ii) the mortality effect of externalities relating to matching and bargaining.

By solving the planner's problem with and without frictions, we find that search frictions unambiguously increase the socially optimal mortality rate. The planner essentially compares the current costs of safety measures with their long-term benefits. The latter accrue from a worker's higher life expectancy, which translates into higher lifetime production and utility. Search frictions cause phases of involuntary unemployment, which reduce lifetime production and utility, and therefore lower the long-term benefits of safety measures. This makes lower safety levels optimal, leading to higher mortality.

If safety measures are not centrally mandated but determined bilaterally between workers and firms, mortality may be even higher due to two externalities. First, private agents may not take into account that a worker dying due to occupational risks is not only lost for its former employer but for the economy as a whole. We observe that whether or not this externality on aggregate labor supply is internalized, depends on the structure of bargaining. Second, even if the labor supply externality is internalized, the mortality rate is still affected by the matching externalities common to the DMP framework. In this regard, any deviation from the Hosios (1990) condition is found to further increase workers' mortality rates. From a policy perspective, we discuss how taxes can be used to internalize the two externalities, and show how to design tax schemes that increase occupational safety, while keeping the potentially resulting loss in aggregate output at a minimum.

The paper proceeds as follows. Section 2 solves the planner's problem for the socially optimal

[2]Throughout the paper, we understand occupational safety as protecting workers against both work-related injuries and work-related diseases.

mortality rate in a frictionless labor market. Section 3 introduces the search frictions and solves the planner's problem once again, before turning to the decentralized economy in Section 4. Section 5 concludes. All mathematical proofs are delegated to the appendix.

2 Frictionless labor market

2.1 Demography and production

To assess the impact of search frictions on mortality, we first solve the social planner's problem in a frictionless labor market. Each period, the planner can freely allocate the mass N of living individuals to employment or unemployment. The mass of employed and unemployed is denoted by L and U, respectively. While unemployed die at an exogenous rate m_U, the mortality rate of employed, m, is endogenously chosen by the planner. Assuming an exogenous mass of newborns B, the population size evolves according to[3]

$$\dot{N} = B - mL - m_U U. \tag{1}$$

Every unemployed generates a flow of home production of $z > 0$. The production of an employed individual is measured in terms of the flow of effective output $y(m)$, which captures output minus the costs of safety measures. These costs can be direct, like regular maintenance of machines or purchasing safety equipment, as well as indirect through lower productivity due to shorter work shifts or time spent on safety routines. The properties of the effective output function are summarized in Assumption 1.

Assumption 1. *For $m \geq 0$, effective output $y(m)$ is twice continuously differentiable and satisfies*

(i) monotonicity and concavity, $y'(m) > 0$, $y''(m) < 0$, with $\lim_{m\to\infty} y'(m) = 0$,

(ii) for some $m > 0$, individuals produce more on a job than at home in present discounted value terms, $\frac{y(m)}{r+m} > \frac{z}{r+m_U}$,

(iii) but this is not the case at $m = 0$, $\frac{y(0)}{r} \leq \frac{z}{r+m_U}$.

By property (i), the current effective output of an employment relation can be increased by allowing higher mortality as this reduces prevention costs. Concavity implies that these output gains become smaller with increasing mortality. Equivalently, reducing mortality becomes more and more costly the lower it already is. This reflects that an initial drop in mortality can be achieved by relatively cheap measures such as buying safety gloves or glasses, while further reductions in mortality require increasingly expensive measures.[4]

[3] We generally omit time indices to simplify notation. We do not model individual mortality as a state variable in order to keep the model tractable.

[4] Since our model is in continuous time, the worker is assumed to finish production $y(m(t))$ before he eventually dies with rate $m(t)$ at the next instant. In discrete time, depending on the duration of a period, a unimodal effective production function may be more realistic, as it allows to capture that the worker may die during the period t production process. In fact, all results of this paper continue to hold if Assumption 1(i) is replaced by the property that $y(m)$ attains a unique maximum at $\overline{m} > 0$ and that it is concave for $m < \overline{m}$.

Property (ii) and (iii) are technical. Essentially, property (ii) guarantees that employment is positive in optimum. Property (iii) ensures that the optimal mortality rate of employed individuals is strictly positive, as reducing the mortality rate to 0 would be too costly, making market production inferior to home production.

2.2 Social planner solution

Assuming that all agents have linear utility, the planner's objective is to find time paths of (m, U, L) that maximize the present discounted value of aggregate output,[5]

$$\int_0^\infty [y(m(t))L(t) + zU(t)]e^{-rt}\, dt,$$

subject to the aggregate population dynamics (1) as well as $L+U = N$ and $U \in [0, N]$. Ignoring the constraint on U for the moment and substituting $L = N-U$, the current value Hamiltonitian of the planner's problem reads

$$\mathcal{H} = y(m)(N - U) + zU + \nu[B - m(N - U) - m_U U],$$

where ν is the costate to N.

Assuming $U < N$, the first order condition with respect to m is

$$\frac{\partial \mathcal{H}}{\partial m} = 0 \quad \Leftrightarrow \quad y'(m) = \nu. \tag{2}$$

By condition (2), the optimal mortality rate equates the marginal gain of mortality in terms of effective output, $y'(m)$, to the marginal cost of mortality, which equals the economic value of a life lost, ν. In an optimum, the latter variable evolves over time according to

$$\frac{\partial \mathcal{H}}{\partial N} = -\dot{\nu} + r\nu \quad \Leftrightarrow \quad \dot{\nu} = (r + m)\nu - y(m). \tag{3}$$

From this point onwards we focus on stationary solutions, $\dot{m} = 0$, which by (2) implies $\dot{\nu} = 0$ and reduces (3) to

$$\nu = \frac{y(m)}{r + m}. \tag{4}$$

Hence the value of a life lost equals the present discounted value of foregone production. Combining this with (2), the optimal mortality rate solves

$$y'(m) = \frac{y(m)}{r + m}. \tag{5}$$

[5] Besides maximizing agents' utilities, a planner could pursue additional goals, such as minimizing the death toll. Such considerations would naturally lead the planner's solution to differ from the decentralized equilibrium, which we therefore abstain from. We also abstract from individuals themselves having non-pecuniary costs of mortality. Extending the model in this direction is straightforward and largely equivalent to appropriately adjusting the effective output function.

It is easy to see that at the optimal mortality rate, the value of a worker given in (4) is maximized. Proposition 1 establishes uniqueness of the planner's solution and verifies that the associated optimal level of unemployment is zero. Correspondingly, the population size is $N = L = \frac{B}{m}$ in steady state.[6]

Proposition 1. *Without search frictions, the social planner's problem has a unique stationary solution with $U^{**} = 0$ and mortality rate $m^{**} > 0$ characterized by* (5)*.*

Condition (5) reveals that the socially optimal mortality rate m^{**} depends on the discount rate r as well as on the effective production function. The higher r, the less the planner values the future output gains relative to the current output costs of occupational safety, and the higher is the optimal mortality rate. To illustrate the dependence on the shape of the production function, assume $y(m) = Am^{\alpha}$ with $A > 0$ and $\alpha \in (0, 1)$. It is easy to verify $m^{**} = \frac{\alpha}{1-\alpha} r$. Hence the tighter the link between mortality and effective output, the higher is the optimal mortality rate. For $\alpha \to 0$, a reduction in mortality has no detrimental effect on output and thus $m^{**} \to 0$. For $\alpha \to 1$, reducing mortality becomes increasingly costly and $m^{**} \to \infty$.[7]

3 Frictional labor market

3.1 Labor flows

From now on, assume that the labor market dynamics are subject to the search and matching frictions typical in the DMP framework. Each period, the mass of unemployed U and the mass of vacancies V are brought together by a constant returns to scale matching function $M(U, V)$. The rate at which vacancies are filled is denoted by $q(\theta) := \frac{M(U,V)}{V} = M(\frac{1}{\theta}, 1)$ where $\theta := \frac{V}{U}$ is the labor market tightness. The rate at which unemployed find a job is $p(\theta) := \frac{M(U,V)}{U} = q(\theta)\theta$, and the elasticity of the matching function with respect to unemployment is $\eta(\theta) := \frac{\partial \ln M(U,V)}{\partial \ln U} = -\frac{q'(\theta)\theta}{q(\theta)}$. These objects satisfy the standard properties of Assumption 2.

Assumption 2. *The job-finding rate $p(\theta)$ and the vacancy-filling rate $q(\theta)$ are continuously differentiable with*

(i) $\lim_{\theta \to 0} p(\theta) = 0$, $\lim_{\theta \to \infty} p(\theta) = \infty$, $p'(\theta) > 0$,

(ii) $\lim_{\theta \to 0} q(\theta) = \infty$, $\lim_{\theta \to \infty} q(\theta) = 0$, $q'(\theta) < 0$,

(iii) $\eta(\theta)$ *is non-decreasing.*

[6] The prediction of full employment is due to our agents being *ex ante* homogeneous. If Assumption 1(ii) applied only to a fraction of the individuals (e.g. due to heterogeneity in z or m_U), the model would feature voluntary unemployment. While this is not the point of the paper, it is interesting to note from (5) that as long as the effective production function does not differ, any employed individual faces the same mortality rate.

[7] Note that the model does not take a stance whether the mortality of employed exceeds the mortality of unemployed. Depending on the parameterization, both outcomes can be achieved. Empirically, mortality rates of unemployed are higher than those of employed workers in most occupations (Paglione et al., 2020).

Everybody is assumed to participate in the labor market, such that $N = L + U$. The population dynamics are governed by the differential equations

$$\dot{L} = -(m+s)L + p(\theta)U, \tag{6}$$

$$\dot{U} = B + sL - (p(\theta) + m_U)U, \tag{7}$$

$$\dot{N} = B - mL - m_U U. \tag{8}$$

The dynamics of the aggregate population (8) are as above. The evolution of the mass of employed and unemployed are described by (6) and (7), respectively. Each period, unemployed find a job at rate $p(\theta)$, while employed move into unemployment at an exogenous rate s. As before, employed individuals die at rate m, while unemployed individuals face an exogenous mortality rate m_U. Newborns start their economic lives without a job.

In a stationary economy with constant inflows, $\dot{B} = 0$, equations (6)–(8) yield

$$L = \frac{p(\theta)}{p(\theta)m + m_U(m+s)}B, \quad U = \frac{m+s}{p(\theta)m + m_U(m+s)}B, \quad N = \frac{m+s+p(\theta)}{p(\theta)m + m_U(m+s)}B.$$

The steady state unemployment rate is $\frac{U}{N} = \frac{m+s}{m+s+p(\theta)}$.

3.2 Social planner solution

If the planner is not bound by the matching frictions and can freely move individuals between employment and unemployment, the analysis is as in Section 2.2. The typical assumption in the matching literature, however, is that the planner cannot overcome the matching frictions and must work through the matching function (Pissarides, 2000, Ch. 8). In contrast to Section 2, the planner then cannot control U directly but only indirectly via creating vacancies V. Assuming a flow cost $c > 0$ per vacancy, the planner maximizes

$$\int_0^\infty [y(m(t))L(t) + zU(t) - cV(t)]e^{-rt}\, dt$$

subject to the population dynamics (6)–(8) as well as $L + U = N$ and $U \in [0, N]$. While the planner essentially chooses time paths for (m, V), it is convenient to reformulate the problem in terms of (m, θ) by writing $V = \theta U$. Furthermore, we substitute $L = N - U$ and omit (6) as well as the static constraint on U from the maximization problem. The current value Hamiltonian then reads

$$\mathcal{H} = y(m)(N-U) + zU - c\theta U + \mu[B + s(N-U) - (p(\theta)+m_U)U] + \nu[B - m(N-U) - m_U U],$$

where μ and ν are the costates to U and N, respectively.

Assuming $0 < U < N$, the first order conditions for an interior optimum read

$$\frac{\partial \mathcal{H}}{\partial m} = 0 \quad \Leftrightarrow \quad y'(m) = \nu, \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial \theta} = 0 \quad \Leftrightarrow \quad c = -p'(\theta)\mu = -(1-\eta(\theta))q(\theta)\mu. \tag{10}$$

Condition (9) coincides with (2), while condition (10) balances the costs of an additional vacancy with the benefits of lower unemployment through increased job-finding. In an optimum, the dynamics of the costate variables are

$$\frac{\partial \mathcal{H}}{\partial U} = -\dot{\mu} + r\mu \quad \Leftrightarrow \quad \dot{\mu} = (r + s + m_U + p(\theta))\mu + y(m) - z + c\theta - \nu(m - m_U), \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial N} = -\dot{\nu} + r\nu \quad \Leftrightarrow \quad \dot{\nu} = (r+m)\nu - y(m) - s\mu. \tag{12}$$

From this point onwards, we again focus on stationary solutions, $\dot{m} = \dot{\theta} = 0$. By the first order conditions, this implies $\dot{\nu} = \dot{\mu} = 0$. Equation (12) gives the economic value of a lost worker as

$$\nu = \frac{y(m) + s\mu}{r+m}. \tag{13}$$

Similar to (4), ν equals the present discounted value of a worker's forgone production in case of death. Yet, it now takes into account that the worker may have become unemployed in the future due to a separation shock. Combining (9) and (13) yields

$$y'(m) = \frac{y(m) + s\mu}{r+m}. \tag{14}$$

Comparing this condition to (5) reveals that while the search frictions do not affect the marginal gain of mortality, they lower its marginal costs, since unemployment reduces a worker's lifetime production in present discounted value terms (note that $\mu < 0$ by (10)). As a result, search frictions increase the optimal mortality rate, see Section 3.3 for further discussion.

Substituting (10) and (13) into (11) to replace c and ν, the steady state value of an additional unemployed becomes

$$\mu = -\frac{(r+m_U)y(m) - (r+m)z}{[r + m_U + p(\theta)\eta(\theta)](r+m) + s(r+m_U)}. \tag{15}$$

The value of μ corresponds to the change in the present discounted value of output if a worker switches from employment to unemployment. In optimum, this is negative by (10), such that frictional unemployment lowers aggregate output.

Substituting (15) back into (10) yields

$$(1-\eta(\theta))\frac{(r+m_U)y(m) - (r+m)z}{[r + m_U + p(\theta)\eta(\theta)](r+m) + s(r+m_U)} = \frac{c}{q(\theta)}. \tag{16}$$

Like in the basic DMP model, this equation determines optimal job creation. To pin down the optimal mortality rate, use (15) to eliminate μ from (14), which after some algebra yields

$$y'(m) = \frac{[r + m_U + p(\theta)\eta(\theta)]y(m) + sz}{[r + m_U + p(\theta)\eta(\theta)](r + m) + s(r + m_U)}. \tag{17}$$

With search frictions, a solution to the planner's problem satisfies (16)–(17).

As in the frictionless economy, it can be shown that the optimal mortality rate maximizes the value of a worker, ν, on the right-hand side of (17). Additionally, it turns out that the optimal mortality rate minimizes μ.[8] This observation is key to our proof of existence and uniqueness of a solution, which solely focuses on the planner's *job creation curve* $\theta^*(m)$ defined by (16). By Lemma 2 in the appendix, this curve is hump-shaped, which reflects that the planner creates fewer vacancies if the mortality of employed workers is very high (as the expected duration of matches is short) but also if mortality is very low (as the required safety measures depress effective output). Since $\mu = -\frac{c}{(1-\eta(\theta))q(\theta)}$ by (10), μ is minimized when the tightness θ is maximized. Hence the planner's solution corresponds to the unique peak of the job creation curve as postulated by Proposition 2. The planner therefore seeks to make the effect of the frictions on the job-finding rate $p(\theta)$ as small as possible.

Proposition 2. *With search frictions, the social planner's solution* (m^*, θ^*) *is unique and corresponds to the peak of the job creation curve* $\theta^*(m)$ *defined by* (16)*.*

This result is graphically illustrated in Figure 1, where JC corresponds to the hump-shaped job creation curve defined by (16). The job destruction curve JD is defined by (17) and downwards sloping. Intuitively, a higher tightness θ increases the job-finding rate and thus μ, as the expected output loss in case of unemployment decreases. By (13), this increases the valuation of a worker's life, ν, and thus the marginal cost of mortality. Therefore, the optimal mortality rate is decreasing in θ along the JD curve. The planner's optimum lies at the intersection of the two curves, which by Proposition 2 coincides with the peak of the JC curve.

3.3 The impact of search frictions on mortality

Since the job-finding rate $p(\theta)$ enters the right-hand side of (17), the presence of labor market frictions affect the optimal mortality rate. Hence, in the presence of search frictions, the solution to the planner's problem m^* is only *constrained efficient*.

The difference in the optimality conditions for mortality (5) and (17) arises from the altered value of ν, which measures the value of a life lost in terms of foregone output. Since workers are more productive in jobs than at home ($\mu < 0$), the presence of frictional unemployment decreases a worker's expected lifetime production and thus decreases the marginal costs of mortality. This implies that the optimal mortality rate is higher in the presence of labor market frictions.

[8]Notice that (17) is equivalent to $\frac{\partial \mu}{\partial m} = 0$ with μ given in (15), and that $\frac{\partial^2 \mu}{\partial m^2} > 0$ at the optimal m.

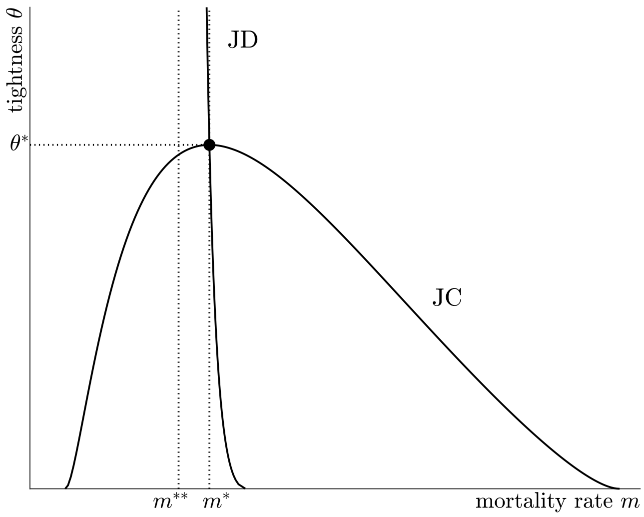

Figure 1. Planner's solution (m^*, θ^*) in the presence of search frictions

The unambiguous increase in mortality was not to be expected, since *ceteris paribus*, a lower m can ameliorate frictional unemployment and reduce the expenditures for vacancy posting. However, Proposition 2 reveals that in the output-maximizing strategy, the planner directly addresses the ultimate source of the welfare losses, which is the depressed job-finding rate. It turns out that the job-finding rate depends on m only through the value of an employed worker.[9] It thus reaches its highest level when ν is maximized, which leads to condition (17).

Proposition 3 shows that the excess mortality caused by the search frictions increases with the severity of the frictions.

Proposition 3. *Let* $\phi := \frac{s}{r+m_U+p(\theta)\eta(\theta)}$. *The constrained efficient mortality rate* m^* *determined by* (17) *is strictly increasing in* ϕ. *For* $\phi \to 0$, *the frictionless mortality rate* m^{**} *given in* (5) *is attained.*

It is straightforward to see from (17) that the optimal mortality rate depends on the labor market frictions only via the fraction given in Proposition 3. With decreasing frictions, $p(\theta) \to \infty$, the fraction approaches zero, such that $m^* \to m^{**}$. *Ceteris paribus*, the excess mortality caused by the frictions is higher, the higher the separation rate and the lower the job-finding rate or the less elastic the matching function responds to changes in unemployment.[10] The fact that $m^* > m^{**}$ is also evident from Figure 1. Proposition 3 implies that the job destruction curve JD approaches the vertical line $m = m^{**}$ for $\theta \to \infty$ and is located right of this line for any finite θ. The intersection with the JC curve must thus necessarily lie to the right of m^{**}.

[9]This is evident from θ being negatively related to μ via (10), and (10)–(12) implying $\mu = -\frac{(r+m_U)\nu - z}{r+m_U+p(\theta)\eta(\theta)}$.

[10]While exogenous in our model, the severity of search frictions may also change the mortality rate of unemployed m_U through their increased risk of long-term unemployment (Browning and Heinesen, 2012). Since the marginal cost of mortality on the right-hand side of (17) is decreasing in m_U, capturing this interaction would amplify the negative effect of search frictions on the provision of occupational safety. Intuitively, a worker's expected lifetime production then not only drops due to the presence of unemployment spells, but also because the mortality experienced while unemployed increases in the expected duration of these spells.

4 Decentralized frictional labor market

Having understood the planner's incentives with and without search frictions, we now decentralize the economy studied in the previous section. Mortality is no longer centrally mandated but bargained between firms and workers together with wages. The attained labor market equilibrium may differ from the planner's solution (m^*, θ^*) due to a range of externalities that are present in the model.

The classical matching externalities may lead the equilibrium tightness to deviate from θ^* (Pissarides, 2000, Ch. 8). This is because an individual firm does not take into account that opening an additional vacancy lowers the vacancy-filling probability of all firms, while on the workers' side, an additional job-seeker reduces the job-finding probability for all other job-seekers. Additionally, our model features an externality that directly affects the mortality rate. As safety measures are bilaterally negotiated between a firm and a worker, the fact that a worker's death not only terminates the current employment relation but permanently lowers the production capacity of the economy is in general not taken into account.

4.1 Value functions

Each firm consists of one job that can either be filled or vacant. Assuming stationarity, the value of a filled and vacant job are, respectively,

$$\begin{aligned} rJ &= y(m) - w - (s+m)(J-V), \\ rV &= -c + q(\theta)(J-V). \end{aligned}$$

A filled job generates a flow profit $y(m) - w$ and is destroyed by an exogenous separation at rate s and by death of the worker at rate m. A vacancy generates a flow cost c and is filled at rate $q(\theta)$. Assuming free market entry of firms, the value of a vacancy is zero in equlibrium, $V = 0$, implying $J = \frac{c}{q(\theta)}$.

The value of employment and unemployment for the worker are, respectively,

$$\begin{aligned} rW &= w - s(W-U) - mW, \\ rU &= z + p(\theta)(W-U) - m_U U. \end{aligned}$$

An employed worker consumes the wage w, moves to unemployment at rate s and dies at rate m. Unemployed consume their home production z, find a job at rate $p(\theta)$ and die at rate m_U. The value of death is zero, since the individual's consumption permanently drops to zero.

4.2 Bargaining

Each period, firm and worker choose a wage w and a mortality rate m that jointly maximize the generalized Nash product

$$\Psi = (W-U)^{\gamma}(J-V)^{1-\gamma},$$

where $\gamma \in (0,1)$ is the bargaining power of the worker.[11] From above, observe that $V = 0$, $J = \frac{y(m)-w}{r+m+s}$, and $W = \frac{w+sU}{r+m+s}$. The value of unemployment U is an equilibrium object and taken as given in the bargaining process.

Assuming $W > U$ and $J > 0$, the first order conditions are

$$\frac{\partial \Psi}{\partial w} = 0 \quad \Leftrightarrow \quad \gamma J = (1-\gamma)(W-U), \tag{18}$$

$$\frac{\partial \Psi}{\partial m} = 0 \quad \Leftrightarrow \quad y'(m) = J + \frac{\gamma J}{(1-\gamma)(W-U)} W. \tag{19}$$

Condition (18) gives rise to the familiar Nash sharing rule, $W - U = \gamma S$ and $J = (1-\gamma)S$ where $S = J + W - U = \frac{y(m)-(r+m)U}{r+m+s}$ is the joint surplus of the match. Substituting this into (19) yields

$$y'(m) = J + W = \frac{y(m) + sU}{r+m+s}. \tag{20}$$

Similarly to the planner's conditions, the left-hand side of (20) measures the marginal benefit of higher mortality in terms of effective output. The right-hand side captures the marginal cost of higher mortality, which in the decentralized economy amounts to losing the joint value of the match, $J + W$. This value comprises the expected output generated on the current job, $\frac{y(m)}{r+m+s}$, and (via U) the worker's expected income earned on future jobs and during unemployment spells. The negotiating parties internalize the labor supply externality if and only if U is such that (20) coincides with (17), compare Section 4.4 for a discussion.

Notice that the bargaining outcome can be interpreted sequentially. Anticipating that each party will receive a fixed share of the joint surplus, m is chosen to maximize S. This is evident from (20) being equivalent to $\frac{\partial S}{\partial m} = 0$, which will be central to the analysis of the equilibrium below.[12]

4.3 Equilibrium

By the Nash sharing rule, the equilibrium value of unemployment satisfies $(r + m_U)U = z + p(\theta)\gamma S$. Substituting this into the definition of S gives equilibrium surplus

$$S = \frac{(r+m_U)y(m) - (r+m)z}{(r+m_U+p(\theta)\gamma)(r+m) + (r+m_U)s}. \tag{21}$$

[11] The view that workers and firms bargain over a compensation package that includes non-wage components, has, for instance, been adopted in Dey and Flinn (2005), where the worker's coverage by health insurance is negotiated together with the wage. We study alternative determination schemes for wages and occupational safety in Section 4.6.

[12] Formally, $\max_{(m,w)} (W-U)^{\gamma} J^{1-\gamma} = \max_m \left\{ \max_w (W-U)^{\gamma} J^{1-\gamma} \right\} = \gamma^{\gamma}(1-\gamma)^{1-\gamma} \max_m S$. In Section 4.6 we discuss the generality of this result.

The corresponding equilibrium value of unemployment is

$$U = \frac{p(\theta)\gamma y(m) + (r+m+s)z}{(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s}. \tag{22}$$

Plugging (21) into the free entry condition, noting $J = (1-\gamma)S$, yields

$$(1-\gamma)\frac{(r+m_U)y(m)-(r+m)z}{(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s} = \frac{c}{q(\theta)}, \tag{23}$$

while substituting (22) into (20) gives, after some algebra,

$$y'(m) = \frac{(r+m_U+p(\theta)\gamma)y(m)+sz}{(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s}. \tag{24}$$

A labor market equilibrium $(\hat{m}, \hat{\theta})$ is characterized by equations (23)–(24). Similar to the planner's solution in Section 3.2, it can be verified that the labor market equilibrium corresponds to the peak of the *job creation curve* $\hat{\theta}(m)$, which is now defined by (23). This is due to the fact that $\hat{m}$ maximizes (21) for $\theta = \hat{\theta}$. Imposing the free entry condition, maximizing (21) is equivalent to maximizing θ along the job creation curve since $S = \frac{c}{(1-\gamma)q(\theta)}$.

Proposition 4. *The equilibrium* $(\hat{m}, \hat{\theta})$ *is unique and corresponds to the peak of the job creation curve* $\hat{\theta}(m)$ *defined by* (23).

Thus, the equilibrium looks qualitatively identical to the planner's solution in Figure 1. To gain further economic insights, let us conduct a small comparative static analysis of the equilibrium with respect to the main model variables. Increasing the slope of $y(m)$ around $\hat{m}$ increases joint surplus (21) and hence $\hat{\theta}(m)$ for $m > \hat{m}$. The peak then moves to the right, resulting in a higher equilibrium mortality rate. The same happens if z or $p(\theta)$ are lowered (for all θ), since surplus increases relatively more for large m. Thus, *ceteris paribus*, higher mortality rates should be observed in jobs in which safety measures are very costly, and for workers whose outside options are poor. As shown in Section 4.4.2, the relationship between equilibrium mortality and the bargaining power γ is not monotonic.

4.4 The impact of externalities on mortality

4.4.1 The labor supply externality

The fact that a diseased worker reduces aggregate labor supply is internalized in the firm-level negotiations if the private costs of mortality equal the social costs of mortality. In this case, conditions (14) and (20) coincide, which proofs equivalent to

$$U = \frac{y(m) + (r+m+s)\mu(m,\theta)}{r+m} \tag{25}$$

where $\mu(m, \theta)$ is the shadow price that a planner assigns to an additional unemployed for a given pair (m, θ). This shadow price can be obtained from (11)–(12) and equals

$$\mu(m, \theta) = -\frac{(r + m_U)y(m) - (r + m)(z - c\theta)}{(r + m_U + p(\theta))(r + m) + s(r + m_U)}. \tag{26}$$

Next, note that free entry and Nash bargaining imply $c\theta = \theta q(\theta)(1-\gamma)S = (1-\gamma)p(\theta)\frac{y-(r+m)U}{r+m+s}$. Substituting this into (26) and plugging the resulting expression into (25) after some algebra yields (22). This proofs that the labor supply externality is internalized in the labor market equilibrium. Even though private agents do not explicitly take into account that a dead worker reduces labor supply on aggregate, in equilibrium this is accurately reflected in the worker's outside option considered in bargaining.

The observation that the labor supply externality is internalized in equilibrium hinges on a particular property of the bargaining scheme proposed in Section 4.2. As mentioned there, the negotiated mortality rate maximizes the *joint* surplus of firm and worker. This is essential, since the production potential outside the firm is only taken into account by the worker, but not by the firm. In Section 4.6 we investigate alternative schemes to determine occupational safety, and their ability to internalize the labor supply externality.

4.4.2 Matching externalities

Even though the labor supply externality is internalized in equilibrium, the mortality rate $\hat{m}$ may differ from the planner's m^*. It may still be distorted by the presence of externalities that arise from the matching process and affect the equilibrium value of U given in (20). Indeed, we observe that the equilibrium conditions (23)–(24) coincide with the planner's conditions (16)–(17) if and only if $\gamma = \eta(\theta)$, which corresponds to the familiar Hosios (1990) condition. In this case, the labor market equilibrium is constrained efficient and attains the mortality rate m^*. Proposition 5 shows that *any* deviation from the Hosios condition increases the mortality rate above its constrained efficient level m^*. Hence both a too low and a too high bargaining power of workers strengthens the negative effects of search frictions on occupational safety.

Proposition 5. *The equilibrium attains the constrained efficient mortality rate m^* if and only if $\gamma = \eta(\theta^*)$. Otherwise, the equilibrium mortality rate exceeds m^*.*

The result of Proposition 5 is illustrated in Figure 2. The relation between bargaining power and equilibrium mortality is U-shaped. While it seems intuitive that workers with little bargaining power (and correspondingly small wage) are willing to take more risk to raise their income, observing the same behavior for workers with high bargaining power is perhaps surprising. It arises from the fact that their high wage reduces their job-finding rate, which reinforces the search frictions and increases the negotiated mortality level. Technically, the equilibrium mortality rate is inversely related to the equilibrium value of unemployment U by (20). Lemma 3 in the appendix shows that U attains its highest value for $\gamma = \eta(\theta^*)$, such that mortality achieves

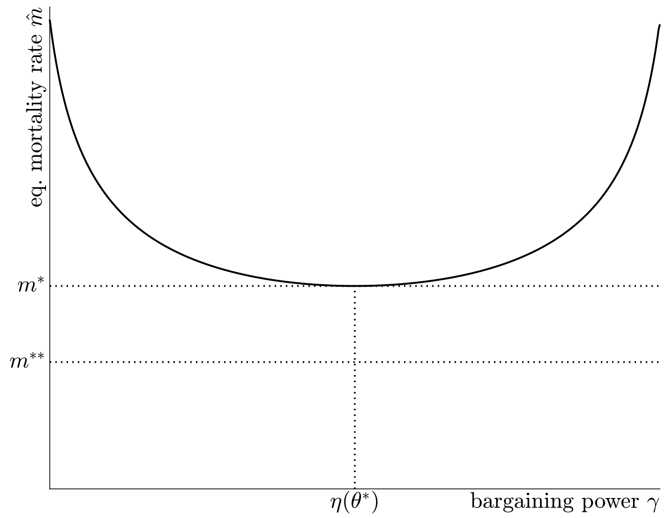

Figure 2. Equilibrium mortality rate $\hat{m}$ as a function of γ

its minimum at this point, where it equals the constrained efficient rate m^*.[13] Even if all externalities are internalized, the mortality rate is still higher than in a frictionless labor market, where it equals m^{**}. While appropriately designed policies can reduce mortality below m^*, this comes with a loss in aggregate output as discussed in the next section.

4.5 Policy

Suppose that there is a government who seeks to maximize aggregate output while keeping the equilibrium mortality rate below some $\bar{m}$. For $\bar{m} \geq m^*$, it is clear from Section 3.2 that the desired pair is the planner's solution (m^*, θ^*). By Proposition 5, this is attained as equilibrium if the Hosios condition is satisfied, such that the government should focus on establishing the right bargaining weights.

For $\bar{m} \geq m^*$, we know from the analysis of Section 3.2 that the optimal mortality rate is $\bar{m}$, and the associated tightness $\bar{\theta}$ lies on the planner's job creation curve (16) illustrated by the solid line in Figure 3. To decentralize $(\bar{m}, \bar{\theta})$ as an equilibrium, we propose a mortality-dependent tax on firms, which changes effective output from $y(m)$ to $y(m) - \Delta(m)$. In equilibrium, all tax revenue is equally distributed among all living individuals by a lump sum transfer t. To pin down the function Δ, let us assume for now that the Hosios condition holds and that the matching elasticity is constant, i.e. $\eta(\theta) \equiv \eta = \gamma$. With the policy, the job creation curve in the decentralized economy (23) becomes

$$(1-\eta)\frac{(r+m_U)(y(m)-\Delta(m)+t)-(r+m)(z+t)}{(r+m_U+p(\theta)\eta)(r+m)+(r+m_U)s} = \frac{c}{q(\theta)}.$$

[13]The property that the equilibrium value of unemployment is maximized under the Hosios condition is inherited from the basic DMP model, see Pissarides (2000, p.187). Endogenous mortality does not destroy this property since $\frac{\partial U}{\partial m} = \frac{\partial S}{\partial m} = 0$ for any given γ.

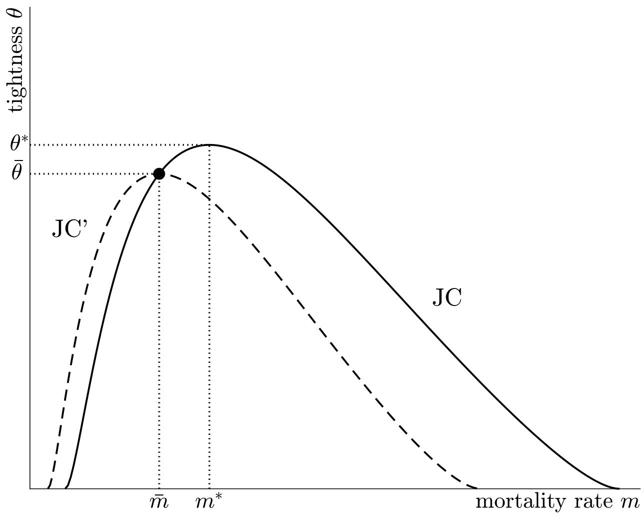

Figure 3. Labor market equilibrium with and without policy

For the equilibrium to lie on the planner's job creation curve (16), the terms arising from the policy must cancel, i.e. $(r + m_U)\Delta(\bar{m}) = (m_U - \bar{m})t$. Additionally, a balanced budget in steady state requires

$$\Delta(\bar{m})L = tN \quad \Leftrightarrow \quad p(\bar{\theta})\Delta(\bar{m}) = (\bar{m} + s + p(\bar{\theta}))t.$$

Combining the two equations reveals $t = \Delta(\bar{m}) = 0$, such that in equilibrium the size of the intervention should be zero. Furthermore, the policy changes equation (24) to

$$y'(m) - \Delta'(m) = \frac{(r + m_U + p(\theta)\eta)(y(m) - \Delta(m) + t) + s(z + t)}{(r + m_U + p(\theta)\eta)(r + m) + (r + m_U)s}.$$

Evaluating this in equilibrium, using $t = \Delta(\bar{m}) = 0$, yields

$$\Delta'(\bar{m}) = y'(\bar{m}) - \frac{(r + m_U + p(\bar{\theta})\eta)y(\bar{m}) + sz}{(r + m_U + p(\bar{\theta})\eta)(r + \bar{m}) + (r + m_U)s}. \tag{27}$$

For $\bar{m} < m^*$, the right-hand side of (27) is positive, implying $\Delta'(\bar{m}) > 0$. Hence, although the tax is zero in equilibrium, the tax schedule is upwards sloping, which increases the marginal cost of mortality. The gradient of the tax schedule must be such that the marginal costs and the marginal benefits of mortality are equalized at $\bar{m}$.

Note that the above conditions only pin down Δ and Δ' at $m = \overline{m}$, but not at other points. The specific shape of Δ in fact does not matter as long as no additional equilibrium arises. This is granted if the altered effective output function $y(m) - \Delta(m)$ satisfies Assumption 1. One tax schedule with this property is

$$\Delta(m) = \lambda[y(m) - y(\bar{m})],$$

with which the government captures a share λ of the production gain that arises from producing with a mortality rate above its target. By construction, $\Delta(\bar{m}) = 0$, while $\Delta'(\bar{m}) = \lambda y'(\bar{m})$.

Substituting this into (27) pins down λ as

$$\lambda = 1 - \frac{(r + m_U + p(\bar{\theta})\eta)y(\bar{m}) + sz}{[(r + m_U + p(\bar{\theta})\eta)(r + \bar{m}) + (r + m_U)s]y'(\bar{m})}.$$

The resulting job creation curve is illustrated by the dashed line JC' in Figure 3. To implement $(\bar{m}, \bar{\theta})$ as an equilibrium, the policy must ensure that the job creation curve of the decentralized economy peaks at this point, compare Proposition 4. While mortality is lower in the new equilibrium, the additional safety measures lead to lower job creation and lower aggregate output (which is maximized at m^*).

If the Hosios condition is not satisfied, $\gamma \neq \eta$, and the government cannot directly affect the bargaining weights, the tax scheme presented above can be modified to take this into account. The required tax is then no longer zero in equilibrium, but accounts for the deviation between γ and η. The tax on firms will be positive if the workers' bargaining power is too low, $\gamma < \eta$. Otherwise, the tax is negative in equilibrium. Since this intervention alters the marginal costs of mortality, the slope of the tax schedule is no longer given by (27), but includes an additional term relating to $\gamma - \eta$.

4.6 Alternative determination schemes for occupational safety

As demonstrated in Section 4.4.1, joint bargaining of wages and safety measures internalizes the labor supply externality in equilibrium. This result holds also if safety levels are determined differently, as long as the outcome maximizes the joint surplus of the match. Consider, for instance, that only the wage is bargained, while m is unilaterally set by the firm *before* the wage negotiation. Since firms anticipate that the joint surplus will be shared according to the Nash rule (18), they solve

$$\max_m J = (1 - \gamma)S \quad \Leftrightarrow \quad \max_m S$$

at the first stage. Therefore, the equilibrium obtained with this bargaining protocol coincides with the equilibrium of Section 4.3. The same applies if m is unilaterally set by the worker or if it were the result of yet another bargain, as long as wages are negotiated afterwards.

Results may change if the level of safety measures is determined *after* wages. To see this, assume that firms can unilaterally choose m after a wage w has been set. Their optimal choice maximizes J subject to $W \geq U$ for the given wage. The first order condition for an interior optimum is

$$y'(m) = J = \frac{y(m) - w}{r + m + s}.$$

The marginal cost of mortality on the right-hand side of this equation is now only the firm's private cost J. The cost of the worker is not taken into account. Comparison with (20) reveals that, irrespective of the wage, mortality is higher than in the equilibrium of Section 4.3.

This observation implies that in the equilibrium of Section 4.3, firms have an incentive to deviate from the negotiated level of m and underinvest into safety measures to increase their profit *ex post*. The bargaining outcome of Section 4.2 thus does not materialize if firms lack commitment and courts cannot verify the implemented level of safety measures. However, a small change in the bargaining setup can avoid *ex post* deviations. Suppose that instead of a pair (m, w), firm and worker negotiate a wage level w as well as a wage gradient w' that specifies the worker's compensation for additional risk-taking. Presented with such a contract, the firm sets m according to the condition $y'(m) - w' = J$, which defines a function $m(w, w')$. It is easy to see that $\frac{\partial m(w,w')}{\partial w'} < 0$, such that a higher wage gradient reduces the optimal mortality rate chosen by the firm. This insight can be used to show that the Nash bargaining problem for (w, w') subject to $m = m(w, w')$ leads to the conditions (18)–(19). These yield a pair (w, m), from which the wage gradient that establishes m as the firm's optimal mortality rate is easily constructed as $w' = y'(m) - J$. This way, the firm's incentive for *ex post* deviations is eliminated, and the equilibrium of Section 4.3 can be attained even if firms can only commit to wage contracts.

The provision of occupational safety may also be inhibited by the classical hold-up problem, which arises when safety measures are implemented *before* wages are set, and part of the safety costs are irretrievable.[14] This changes a firm's threat point in the bargain as it would incur a loss if the worker walked away. Assuming sunk costs $d(m) > 0$, the bargaining problem becomes

$$\max_{w} \, (W - U)^{\gamma}(J + d(m))^{1-\gamma},$$

since the firm's outside option is now $-d(m)$. The solution implies $J = (1 - \gamma)S - \gamma d(m)$. Assuming that the firm unilaterally chooses the level of safety measures before wages are negotiated, the first order condition for m is $\frac{\partial S}{\partial m} = \frac{\gamma}{1-\gamma} d'(m)$. If higher safety measures require more upfront costs, $d'(m) < 0$, the firm chooses a point on the downwards sloping part of the surplus curve, $\frac{\partial S}{\partial m} < 0$. Therefore, joint surplus is no longer maximized. Furthermore, the optimality condition for m can be written

$$y'(m) - \frac{\gamma}{1-\gamma} d'(m)(r + m + s) = \frac{y(m) + sU}{r + m + s},$$

which shows that the firm's marginal gain of mortality increases because part of the additional expenditures on safety measures cannot be shared with the worker. Even if $d'(m) = 0$, the presence of sunk costs affects the mortality rate via U, as the worker effectively receives a higher share in surplus. In any case, a policy along the lines of Section 4.5 can be used to redistribute a part of the firm's upfront expenditures to the households to restore efficient safety provision.

While in a bargaining setting, hold-up and matching externalities can lead to suboptimal equilibrium outcomes, efficiency may always arise if the labor market is organized differently. Suppose that firms post and commit to contracts (m, w), to which workers apply in the manner of directed search (Moen, 1997; Wright et al., 2021).[15] The directed search equilibrium can be

[14] See Malcomson (1997) for a summary of this literature.

[15] If the actual level of job security provided by the firm is not verifiable by a court, the contracts can equivalently be written over (w, w') similar to the discussion above.

characterized as solution to

$$\max_{(m,w,\theta)} p(\theta)(W - U) \quad \text{s.t.} \quad q(\theta)J = c.$$

It is easy to verify that the equilibrium conditions boil down to planner's conditions (16)–(17). Additionally to internalizing the externalities, directed search gets around the hold-up problem (Acemoglu and Shimer, 1999).

5 Conclusion

This paper studied the provision of occupational safety in a labor market with search frictions. To this purpose, the basic Diamond-Mortensen-Pissarides model was extended for mortality shocks with endogenous arrival rate. The presence of search frictions was found to increase the socially optimal mortality rate by lowering safety levels. While the marginal costs of safety measures are unaffected by the frictions, periods of involuntary unemployment decrease a worker's expected lifetime production and utility, and hence the long-run gains of safety measures.

In a decentralized setting, externalities related to matching and bargaining may lead to a further increase in mortality. Exploring a wide scope of determination schemes for wages and occupational safety, we found that the negotiating parties generally internalize the labor supply externality, i.e. the effect of a higher mortality rate on aggregate labor supply. This is far from obvious, since none of the parties explicitly takes the aggregate effects of their decisions into account. Yet, in equilibrium, the worker's outside option turns out to reflect the correct "price" of mortality. Even if the labor supply externality is internalized, the Hosios (1990) condition is required for the equilibrium mortality rate to equal the rate chosen by a planner who is constrained by the search frictions. Any deviation from the Hosios condition leads to higher mortality due to a further drop in workers' expected lifetime production.

Policy initiatives like those of the European Commission and the US government aim to increase occupational safety. This can be welfare improving if the social costs of work-related injuries and diseases exceed the costs considered by private agents. In our model, the only distortions of private incentives were due to labor supply and matching externalities, which could be avoided by giving workers an appropriate bargaining weight in firm-level negotiations. In practice, additional factors such as asymmetric information, cognitive biases, and other externalities may further distort the private provision of occupational safety (Pouliakas and Theodossiou, 2013). While suitable to correct these distortions, policies that focus primarily on occupational safety seem much less suited to address the excessive mortality caused by the search frictions themselves. As we demonstrated, once all externalities have been internalized, a further reduction in mortality inevitably lowers aggregate economic output and hence welfare. To ameliorate the detrimental mortality effects of search frictions, these must be addressed more directly. Accelerating the matching of unemployed to job openings, for example, at the same time increases equilibrium safety levels and aggregate output. Along these lines, the rise in long-term unemployment resulting from the pandemic may inhibit the success of the recent policy initiatives to boost occupational safety if labor market frictions remain elevated.[16]

The model presented in this paper was purposefully kept simple to identify the main mechanisms that affect the provision of occupational safety in a labor market with search frictions. We believe that these mechanisms will remain of central importance in more complex versions of the model. Indeed, our model is general enough to be extended in many directions. For instance, premature death of a worker is the most extreme implication of low occupational safety. Many adverse economic effects already occur during the worker's lifetime in the form of chronic diseases or permanent disability.[17] In modern welfare states, a big chunk of health expenditures are born by the public and are thus not reflected in private decision-making. This creates an externality absent in the presented model. Furthermore, we abstracted from modeling education, which ultimately determines the characteristics of an individual's potential jobs. Distortions in the provision of occupational safety are likely to distort schooling decisions and occupational choices as well. We also neglected life-cycle features. Individual attitudes towards health hazards may vary over a worker's lifetime depending on age, health, and socioeconomic factors. This may call for policies targeted at particular subpopulations. These and further questions are left for future research.

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[16]Compare the blog entry by Pissarides (2020) on the potential persistent increase in long-term unemployment due to the COVID-induced acceleration of automation.

[17]The European Agency for Safety and Health at Work (2017a) calculates that fatal and non-fatal work-related injuries and diseases account for an approximately equal share of GDP loss.

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A Mathematical appendix

This section contains auxiliary results and proofs to the propositions stated in the main text.

A.1 Auxiliary results

Lemma 1. *The function* $\phi(m) := \frac{y(m)}{r+m}$ *satisfies* $\lim_{m\to\infty} \phi(m) = 0$. *It is unimodal with a single peak* $\overline{m} > 0$*, which satisfies* $\frac{y(\overline{m})}{r+\overline{m}} > \frac{z}{r+m_U}$.

Proof. Assumption 1(i) implies $\lim_{m\to\infty} \phi(m) = 0$ by L'Hopital's rule. The derivative is $\phi'(m) = \frac{1}{r+m}[y'(m) - \frac{y(m)}{r+m}]$. At any point that satisfies $\phi'(m) = 0$, the second derivative is $\frac{y''(m)}{r+m} < 0$. Hence any local optimum of ϕ is a maximum. Assumption 1(ii) guarantees an $\tilde{m} > 0$ such that $\phi(\tilde{m}) > \frac{z}{r+m_U} \geq 0$. As ϕ asymptotically approaches 0, it either has a single peak $\overline{m} > 0$ or is monotonically decreasing. The latter is ruled out by Assumption 1(iii), which implies $\phi(0) < \phi(\tilde{m})$. Finally, $\phi(\overline{m}) \geq \phi(\tilde{m}) > \frac{z}{r+m_U}$, since $\overline{m}$ maximizes ϕ. □

Lemma 2. *Equation* (16) *defines a function* $\theta^*(m)$ *with the following properties:*

(i) the domain of θ^* is a non-empty interval $M = (\underline{m}, \overline{m}) \subset \mathbb{R}^+$ whose boundaries satisfy $\frac{y(m)}{r+m} = \frac{z}{r+m_U}$,

(ii) the sign of $\frac{d\theta^*}{dm}$ is the opposite of $\frac{\partial \mu}{\partial m}$, where μ is given in (15)*,*

(iii) the function is unimodal with a single peak and approaches zero at the boundaries of M.

Proof. Property (i): Since $q(\theta)$ is positive for any finite θ by Assumption 2(ii), a solution to (16) can only exist if $\mu < 0$, which requires $m \in M := \{m \geq 0 : \frac{y(m)}{r+m} > \frac{z}{r+m_U}\}$. On the other hand, for any $m \in M$, the properties of Assumption 2 ensure that (16) has a unique solution $\theta^*(m) > 0$. Hence the domain of θ^* is M, which by Assumption 1(iii) does not include zero. The unimodality result of Lemma 1 implies that M is a non-empty open interval.

Property (ii): Applying the implicit function theorem to (16) gives

$$\frac{d\theta^*}{dm} = \frac{-(1-\eta(\theta))\frac{\partial \mu}{\partial m}}{(1-\eta(\theta))\frac{\partial \mu}{\partial \theta} - \eta'(\theta)\mu - \frac{c}{q(\theta)^2}q'(\theta)}$$

with μ given in (15). Since $\mu < 0$, $\frac{\partial \mu}{\partial \theta} > 0$, and Assumption 2, the denominator is strictly positive. Furthermore, $1 - \eta(\theta) = \frac{p'(\theta)\theta}{p(\theta)} > 0$, such that the sign of $\frac{d\theta^*}{dm}$ equals the sign of $-\frac{\partial \mu}{\partial m}$.

Property (iii): At the boundaries of M, $\frac{y(m)}{r+m} \to \frac{z}{r+m_U}$ and thus $\frac{c}{q(\theta)} \to 0$. By Assumption 2(ii), this implies $\theta \to 0$. Since $\theta(m) > 0$ for $m \in M$, θ must attain a local maximum on M. This maximum is unique provided that no inner local minimum exists. Property (ii) of this Lemma implies that $\frac{d\theta^*}{dm} = 0$ if and only if $\frac{\partial \mu}{\partial m} = 0$. Every such point is a local maximum of θ^*, since $\frac{d^2\theta^*}{dm^2}$ becomes proportional to $-\frac{\partial^2 \mu}{dm^2} = \frac{ry''(m)}{(r+m_U+p(\theta)\eta(\theta))(r+m)+s(r+m_U)} < 0$. By continuity, $\frac{d\theta^*}{dm}$ cannot change sign more than once, such that the maximum is unique. □

Lemma 3. *The equilibrium value of unemployed U is unimodal in γ and peaks at $\gamma = \eta(\theta)$.*

Proof. Differentiating (22) with respect to γ gives $\frac{dU}{d\gamma} = \frac{\partial U}{\partial m}\frac{dm}{d\gamma} + \frac{\partial U}{\partial[p(\theta)\gamma]}\frac{d[p(\theta)\gamma]}{d\gamma}$. It is straightforward to show $\frac{\partial U}{\partial m} = 0$ and $\frac{\partial U}{\partial[p(\theta)\gamma]} = \frac{r+m+s}{(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s}S$. Hence the sign of $\frac{dU}{d\gamma}$ coincides with the sign of $\frac{d[p(\theta)\gamma]}{d\gamma}$, which is shown to equal the sign of $\eta(\theta) - \gamma$ in the proof of Proposition 5. The rest of the proof is analogous to there. □

A.2 Proofs of propositions

Proof of Proposition 1. We first show that (5) defines a unique mortality rate. Note that (5) corresponds to the first order condition of $\max_m \frac{y(m)}{r+m}$. By Lemma 1, the objective function is unimodal with a single peak, such that (5) is satisfied by exactly one m.

Next, note that (5) was obtained assuming $U < N$. The maximized value of the Hamiltonian is $\mathcal{H}^{**} = \frac{ry(m^{**})}{r+m^{**}}(N-U) + zU + \frac{y(m^{**})}{r+m^{**}}[B - m_U U]$. To determine the optimal value of U, observe $\frac{\partial \mathcal{H}^{**}}{\partial U} = z - \frac{r+m_U}{r+m^{**}}y(m^{**})$. As m^{**} maximizes $\frac{y(m)}{r+m}$, the derivative is strictly negative by Lemma 1. Therefore, $U^{**} = 0$, and the initial assumption is satisfied. □

Proof of Proposition 2. By Lemma 2, $\theta^*(m)$ has a unique peak characterized by $\frac{\partial \mu}{\partial m}$ = where

$$\frac{\partial \mu}{\partial m} = \frac{r+m_U}{[r+m_U+p(\theta)\eta(\theta)](r+m)+s(r+m_U)}\left\{\frac{[r+m_U+p(\theta)\eta(\theta)]y(m)+sz}{[r+m_U+p(\theta)\eta(\theta)](r+m)+s(r+m_U)} - y'(m)\right\}.$$

Hence the point (m, θ) that maximizes $\theta^*(m)$ solves the planner's problem because it satisfies (16)–(17). On the other hand, any solution satisfies $\frac{\partial \mu}{\partial m} = 0$ and therefore corresponds to an interior extremum of $\theta^*(m)$. Since $\theta^*(m)$ is unimodal, the only interior extremum is the unique global maximum. □

Proof of Proposition 3. Equation (17) can be rewritten $y'(m) = \frac{y(m)+\phi z}{r+m+\phi(r+m_U)}$. For $\phi = 0$, the condition simplifies to (5). The implicit function theorem yields $\frac{dm}{d\phi} = \frac{\mu}{y''(m)[r+m+\phi(r+m_U)]}$, which is positive since $y''(m) < 0$ and $\mu < 0$. □

Proof of Proposition 4. The result immediately follows from Lemma 2 and Proposition 2 by setting $\eta(\theta) = \gamma$ and noting $\mu = -S$. □

Proof of Proposition 5. I first verify that like in the basic DMP model, the equilibrium tightness is strictly decreasing in γ. Consider the total derivative of (23),

$$\left[(1-\gamma)\frac{\partial S}{\partial \gamma} - S\right]d\gamma + \left[(1-\gamma)\frac{\partial S}{\partial \theta} + c\frac{q'(\theta)}{q^2(\theta)}\right]d\theta + (1-\gamma)\frac{\partial S}{\partial m}dm = 0,$$

where all expressions are evaluated in equilibrium and S is given in (21). Since m maximizes S, the last term is zero and evaluating the remaining terms yields

$$\frac{d\theta}{d\gamma} = -\frac{(r+m_U+p(\theta))(r+m)+(r+m_U)s}{[p(\theta)\gamma+\eta(\theta)(r+m_U)](r+m)+\eta(\theta)(r+m_U)s}\cdot\frac{\theta}{1-\gamma} < 0.$$

Second, observe from (24) that the equilibrium mortality rate depends on γ only via the joint term $p(\theta)\gamma$. The implicit function theorem reveals

$$\frac{\partial m}{\partial[p(\theta)\gamma]} = \frac{y(m)-y'(m)(r+m)}{y''(m)[(r+m_U+p(\theta)\gamma)(r+m)+(r+m_U)s]} < 0.$$

The sign follows from $y'' < 0$ and substituting (24), by which $y(m) > y'(m)(r+m)$. Furthermore,

$$\frac{d[p(\theta)\gamma]}{d\gamma} = p(\theta) + p'(\theta)\gamma\frac{d\theta}{d\gamma} = p(\theta)\left[1+(1-\eta(\theta))\frac{d\theta}{d\gamma}\frac{\gamma}{\theta}\right].$$

Substituting $\frac{d\theta}{d\gamma}$ from above and collecting terms yields

$$\frac{d[p(\theta)\gamma]}{d\gamma} = p(\theta)\frac{(r+\gamma p(\theta))(r+m)+rs}{[\gamma p(\theta)+\eta(\theta)r](r+m)+\eta(\theta)rs}\frac{\eta(\theta)-\gamma}{1-\gamma}.$$

Putting things together, the sign of $\frac{dm}{d\gamma} = \frac{\partial m}{\partial[p(\theta)\gamma]}\frac{d[p(\theta)\gamma]}{d\gamma}$ equals the sign of $\gamma - \eta(\theta)$. Since $\frac{d\theta}{d\gamma} < 0$ and $\eta' \geq 0$, it follows that $\gamma - \eta(\theta)$ is strictly increasing in γ. Therefore, m has a unique minimum, which satisfies $\gamma = \eta(\theta)$. □