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Future research on urban transportation network modeling

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Abstract

The distribution of citations of five path-breaking publications across regional science and transportation journals is examined to gauge whether RSUE was a significant player in the evolution of urban transportation network modeling research. Concluding this research area is pertinent to the journal, and research advances in transportation network equilibrium modeling are traced through five phases of development from its beginnings in the early 1950s. Then, the future of this research area is examined with regard to four issues: representation of congestion; tour-based representation of travel; non-separable travel time–flow relationships; and dynamic network flows.
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1. Introduction and background

Forty years ago was evidently a very good time to launch new journals in regional science and closely related fields such as transportation and quantitative geography, as it was then known. To begin, in 1967 three academic journals in transportation were founded: *Transportation Research*, *Transportation Science*, and the *Journal of Transport Economics and Policy*. The *Annals of Regional Science* also began publication in 1967, followed by *Environment and Planning* and *Geographical Analysis* in 1969, and *Regional and Urban Economics* in 1971. The last was reorganized as *RSUE* in 1975, and is now celebrating its 35th anniversary with this special issue. Other regional science journals were founded earlier, of course: *Papers of the Regional Science Association* in 1955, reorganized as *Papers in Regional Science* in 1991, and *Journal of Regional*

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Science in 1958. Other journals with “regional science” or closely related terms in their names were initiated by various sections of the Regional Science Association during the 1970s. For a listing of current journals now deemed to be related to Regional Science, visit www.regionalscience.org/Resources/Journals.htm.

The proliferation of journal publications that began in the 1960s in all fields has led to the current situation described in the Web of Science Databases (www.thomson.com). The *Science Citation Index Expanded* consists of a multidisciplinary index to the journal literature of the sciences. It fully indexes 5900 major journals across 150 scientific disciplines from 1955 to the present. The *Social Science Citation Index* presently indexes 1725 journals across 50 social science disciplines from 1956 to the present, as well as individually selected, relevant items from over 3300 of the world’s leading scientific and technical journals.

In considering the future of urban transportation network models, that slice of regional science of particular interest and relevance to me, I initially wondered whether this topic is sufficiently central to regional science journals generally, and RSUE in particular, to be appropriate for this forum. To address this question, in mid-2006 I used the *Web of Science* to examine the role of regional science journals in publishing articles central to my research over the past 40 years. To obtain a sense of which journals have published articles of importance to me, I undertook a search of citations of the following four early publications:

Beckmann, M., McGuire, C. B., and Winsten, C. B., 1956. Studies in the Economics of Transportation, Yale University Press, New Haven, CT; first issued in 1955 as RM-1488 by the Rand Corporation, Santa Monica, CA.

Herbert, J. D., Stevens, B. H., 1960. A Model for the Distribution of Residential Activity in Urban Areas, Journal of Regional Science 2, 21–36.

Lowry, I. S., 1964. A Model of Metropolis, RM-4035-RC, Rand Corporation, Santa Monica, CA.

Wilson, A. G., 1967. A Statistical Theory of Spatial Distribution Models, Transportation Research 1, 253–269.

Table 1
Number of citations of five classic works by journal and field

Publication:	Beckmann	H–S	Lowry	Samuelson	Wilson	Total
Regional Science journals						
Annals of Regional Science	4	0	5	13	4	26
International Regional Science Review	0	0	1	2	0	3
Journal of Regional Science	9	11	12	30	18	80
Journal of Urban Economics	1	10	4	2	1	18
Papers in Regional Science	3	3	5	9	5	25
Regional Science and Urban Economics	6	5	4	15	9	39
Geography journals						
Environment & Planning — all parts	15	33	67	13	87	215
Geographical Analysis	2	1	6	7	21	37
Transportation journals						
J. of Transport Economics & Policy	7	1	2	1	3	14
Transportation Research — all parts	95	7	14	19	59	194
Transportation Science	40	1	1	13	1	56
Total citations in above journals	182	72	121	124	208	707
Total citations in all journals	357	140	305	495	311	1608

As a further basis for comparison, I added the following article, because of its relationship to the field of transportation network models, and to papers published in RSUE:

Samuelson, P.A., 1952. Spatial Price Equilibrium and Linear Programming, American Economic Review 42, 283–303.

The tabulations compiled from the Web of Science are shown in Table 1.

Before drawing any conclusions from these tabulations, several comments seem appropriate:

1. The number of citations should not be compared strictly, since the duration of publication and total number of papers published by each journal is quite different; moreover, no distinctions are made with regard to trends over calendar time.
2. Two journals, IRSR and JUE, are included mainly for comparative purposes.
3. Considering regional science journals, the principal journals citing these five works are clearly JRS and RSUE.
4. The four multidisciplinary journals making up E&P constitute the largest single source of citations. These are classified here as "geography," although they may be regarded as broader in scope, especially in their early years of publication.
5. The three transportation journals have the largest group of cited papers for these five transportation-oriented works. Of these, the five parts of TR have the largest share, although Part B tends to dominate the others.

The principal conclusion that I believe should be drawn from these tabulations is that all three groups of journals have played a significant role in publishing research related to the five selected works. Clearly, the transportation journals, *Transportation Research* and *Transportation Science*, are the most important publishers of articles citing these works, followed by *Environment and Planning*. RSUE is better represented in citations of these five works than I guessed, especially since its citations were only tabulated by the Web of Science from 1975 onwards, as compared to JRS, which began in 1958. Thus, I am satisfied that the focus of my essay is indeed relevant to RSUE.

With this factual background in hand, I now turn to an examination of how one of these fields developed. In the following review, I emphasize the role of timing and chance, realizing that science does not proceed in a steady progression of ideas in response to a plan. Then I briefly examine four research problem areas that may be ripe for new advances.

2. What is past is prologue[1]

The objective of this piece is to present a short, crystal-ball gazing essay speculating on an aspect of the future development of the field, in this case urban transportation network modeling. As a starting point for my "gazing," I examined the phases of research advances in the area of transportation network equilibrium, a topic whose history is best known to me among the five works examined above. This consideration led to the following list of research phases:

1. initial formulation of the problem, often in a simplified form;
2. creation and testing of convergent algorithms for solving the problem;

[1] W. Shakespeare.

3. use of solution algorithms to address realistic and important policy questions;
4. generalizations, extensions and refinements of the initial problem formulation;
5. ongoing reformulations of the problem, solution algorithms, analyses of model properties, and applications.

Martin Beckmann, Bartlett McGuire and Christopher Winsten (1956) formulated what we now know as the transportation network user-equilibrium problem with variable demand, as well as the associated transportation network system-optimal problem with variable demand, during late 1953 and early 1954, thereby providing an analytical construct for the verbal criteria of Wardrop (1952). The user-equilibrium problem formulation they stated was completely novel, both substantively and methodologically, in that they proposed the first general mathematical formulation of the problem by applying the newly proven Kuhn–Tucker conditions. They analyzed the model's properties, and solved a toy example, but did not propose a solution algorithm. Although their book was extensively reviewed, no reviewer identified the significance of the problem formulation, which now may be regarded as having launched an entire subfield of transportation research. Subsequently, a few authors independently formulated the related problem of transportation network user-equilibrium with fixed demand (Charnes and Cooper, 1958, 1961; Jorgensen, 1963; Braess, 1968).

Although Beckmann et al. (1956) was widely circulated, no algorithms were proposed to solve their formulation, or the related fixed demand version, until the late 1960s. Then, within a relatively short period, 1968–1973, several algorithms were identified, analyzed and solved computationally (Dafermos, 1968; Bruynooghe et al., 1969; Murchland, 1970; Evans, 1973; Florian et al., 1975; LeBlanc, 1973; Nguyen, 1973). Out of these four Ph. D. theses and three papers emerged the linearization method, based on Frank and Wolfe (1956), the algorithm presently found in several commercial travel forecasting software systems. Evans examined the user-equilibrium problem with variable demand in depth, leading to her proposal for a partial linearization algorithm with a rigorous proof of convergence. Murchland and Florian et al. offered related algorithms for the variable demand problem, but these were not found to be computationally efficient (Frank, 1978).

The application of travel forecasting methods by practitioners proceeded in parallel with the more rigorous formulation of theoretical models from 1953 onwards. Often the early versions, known as trip distribution and traffic assignment methods, were simply heuristic procedures for computing forecasts that implied simple assumptions such as the gravity hypothesis and the use of shortest routes through a network. Over time, these procedures were replaced by well-formulated models and convergent solution algorithms that can be proven to solve the problem of interest. The procedure for solving the resulting sequence of models, however, was not displaced by the original, integrated formulation of Beckmann et al., as specialized by Murchland, Evans and Florian to urban transportation planning applications. Consequently, a method for solving the sequential, or four-step, procedure in a consistent manner with regard to travel times and costs has yet to be widely adopted by practitioners (Boyce and Bar-Gera, 2006).

Following the initial round of academic algorithmic research, and applications, promising reformulations and extensions of the original problem of Beckmann et al appeared at the end of the 1970s (Aashtiani, 1979; Smith, 1979; Dafermos, 1980). These papers showed how their basic formulation could be generalized using concepts from equilibrium analysis based on fixed point, nonlinear complementarity and variational inequality methods.

Concurrently, several researchers began to synthesize the network equilibrium formulation of Beckmann with the travel choice and urban location models of Herbert and Stevens (1960), Wilson (1967), McFadden (1973) and Williams (1977). An early example is my synthesis of the residential location model of Wilson (1970, pp. 72–77) with the network equilibrium approach, thereby making generalized travel costs endogenous to the model (Boyce, 1980). Los (1979) offered a more detailed version of this approach, and thoroughly explored the economic interpretations of the model. Kim (1983) extended this approach to intraurban commodity flows. Anas (1985) transformed the model into the utility maximizing framework. Oppenheim (1993) extended the integration of models to include goods movements. Yang and Meng (1998) proposed a similar model from a stochastic user-equilibrium viewpoint. Boyce and Mattsson (1999) considered optimal road tolls in a housing location model with user-equilibrium travel costs, extending the housing location model of Anderstig and Mattsson (1991). Nagurney and Dong (2002a,b) offered recent syntheses of user-equilibrium travel choice models, including telecommuting, with urban location models of residential and employment location choice in a variational inequality formulation. For many additional contributions one may consult the reference lists of the above papers.

Attempts to understand and synthesize the classes of problems reviewed above continued through the 1980s. Michael Patriksson, a graduate student in optimization at the Linköping Institute of Technology, Sweden, became intensely interested in these problems. In the first paper of his Licentiate thesis, Patriksson (1991) provided an extensive survey and synthesis of the field initiated by Beckmann and his coauthors. Following the completion of his Ph. D. thesis, Patriksson (1994) published an expanded version of his research as *The Traffic Assignment Problem — Models and Methods*. This comprehensive work, with its 1020 references, provided the first formal treatment of the transportation network equilibrium problem and its solution.

In a departure from the linearization, link-based approach of the early 1970s, Patriksson also devised a route-based algorithm based on the principle of simplicial decomposition for the fixed and variable user-equilibrium problem formulations (Larsson and Patriksson, 1992; Lundgren and Patriksson, 1998). A related algorithm for the fixed demand case was proposed somewhat earlier by Bothner and Lutter (1982), and is found in one practitioner software system.

Nearly ten years after Patriksson became interested in this research problem, another graduate student with training in mathematics, Hillel Bar-Gera, was challenged by the problem of finding a fast and precise solution to the network user-equilibrium problem. He conjectured that an intermediate approach between the aggregate link-based approach and the detailed route-based approach might have potential, which led to his origin-based assignment algorithm (Bar-Gera, 1999, 2002) the first algorithm to achieve convergence to the precision of the computer for large networks. An extension of this algorithm to solve the variable demand user-equilibrium problem was offered by Bar-Gera and Boyce (2003). Subsequently, Bar-Gera devised an algorithm for finding the most likely route flows, given the uniquely-determined user-equilibrium link flow solution (Bar-Gera, 2006).

Given this fast-paced tour of the 50-year history of the field, one might ask to what extent the benefits of past research are being reaped in research as well as planning and policy analysis. The answer, unfortunately, is somewhat mixed. Although the original integrated travel and route choice model formulated by Beckmann is generally not applied in professional practice, it has been implemented and validated with multiple classes for large-scale networks (Lam and Huang, 1992; de Cea and Fernández, 2001; Boyce and Bar-Gera, 2003). Solution of large-scale models of urban travel and location are mainly confined to metropolitan planning agencies. There, sequential or four-step, solution procedures devised by practitioners, and now typically solved with feedback, are generally applied, often without a full understanding of their solution properties (Boyce and Bar-Gera, 2004; Boyce and Williams, 2005). Solution algorithms for the fixed demand user-equilibrium problem were incorporated into several commercial software systems during the past twenty years, and have now largely replaced earlier heuristic methods.

One lesson to be learned from this history is that the evolution of a research field is highly unpredictable. If at any point after the original problem formulation in 1954, informed scholars had offered an opinion on the future development of the area, and the application of these methods to planning and policy analysis, they almost certainly would have badly misjudged the future. Unexpected results, often coming in the form of innovative Ph.D. theses, have contributed much to the field. Still, difficult research problems remain; some are surveyed in the final section of this note.

3. On the future of urban transportation network modeling

My view of urban and regional economics is from the perspective of transportation research generally, and large-scale models of location and travel choice with endogenous travel costs in particular. To what extent is this modeling approach suited to all research and planning needs of readers of RSUE? Small scale models with fixed or user-equilibrium travel costs clearly have their place in journals, textbooks and the classroom for pedagogical purposes; several insightful examples are to be found in papers by Arnott, de Palma and Lindsay, such as Arnott et al. (1990). Insights can be gained, and the direction of responses to changes in road and transit prices can often be inferred. Nevertheless, to understand the network implications of pricing and congestion, larger-scale models appear to be useful, and perhaps necessary.

Formulation, estimation, validation and application of a large-scale location and travel model involve many compromises pertaining to representation of spatial and temporal detail, number of market segments, and the transportation network itself, as well as the constraints of data. Once estimated and solved computationally, the detail of the forecast is often quite overwhelming, and the reasons for the overall pattern of location and travel choices represented in the solution may be more murky than transparent.

Although computers have expanded enormously in speed, memory and disk storage, I believe that achieving a suitable balance among spatial and temporal detail, representation of myriad location and travel choices and computational tractability will occupy researchers for many years to come. Moreover, these questions of model design have not been carefully addressed in the past, often as a result of the urgency of producing computational results in both research and practice.

Despite or perhaps because of these challenges, I would argue that more scholars should add a large-scale modeling activity to their research agenda. Construction and maintenance of a model of location and travel choice for one's home region, or other region of special interest, should be one of the elements in the toolkits most scholars. While not the method of choice for every question that may arise, it should represent an option for some, and ought to be an available research direction for interested graduate students.

Without a doubt, all models, whether large or small, are simplifications of reality. Removing or reducing those simplifications constitute one of the main directions of future research. If a model enhancement is attempted for a particular application, its design must be carefully considered, not only to achieve the objectives being sought by the relevant stakeholders, but also with a view to model properties, resource availability, requirements for use and knowledge deficiencies. In the following, I speculate on some research advances needed to improve the reality of this class of models with regard to their transportation dimension, and to enhance their application for planning and policy analysis. In the following four points, I do not consider location choice aspects specifically, since I have not recently read this literature in detail.

1. The improved representation of traffic congestion in network user-equilibrium models, as well as the means for addressing it (pricing, regulations), clearly ought to be the focus of future research, if models are to be useful, as well as meaningful representations of reality. Adequate representation of congestion, or equivalently the effect of congested travel times and costs on travel and location choices, is only meaningful if represented consistently for all such choices. Moreover, the model must be solved in a manner that the choices affected by congestion are consistently constrained.
   An underlying issue in this regard is the geographic scale of the model implementation and associated data requirements. GIS platforms have revolutionized the handling and representation of spatial data to the extent that the use of aggregate zone systems is sometimes questioned. Sometimes forgotten in such discussions are the properties of the models themselves, which for sound reasons are defined in terms of continuous, real-valued variables. Such model properties are sometimes overlooked in requests for finer and finer model detail.
2. Related to the representation of congestion is the question of tour-based representation of travel as an alternative to trip-based representations, during the period of travel, such as the morning or evening commuting period. Unless travel is defined in a meaningful manner as a sequence of trips, or as travel to a primary destination activity (e.g. from home to work or school) with associated stops en route, then an adequate representation of the effects of travel on network congestion is unlikely to be realized. Unfortunately, this data-related question is often ignored by theoreticians who may tend to accept traditional problem statements without seriously questioning them. Expanding the trip-based format to include tours is not conceptually difficult, but may be computationally awkward. New ways of representing spatial interactions may be the key to solving this problem (Maruyama and Sumalee, 2007).
3. At a more detailed, but also crucial, level is the question of non-separable travel time — flow functions (also known as volume-delay functions). Separable functions, in which the link delay depends only on the link's own flow, have for too long dominated both theory and practice of urban transportation network modeling. The implications of using non-separable functions are well known: non-uniqueness properties of the standard model due to asymmetries in the Jacobian of the link cost function, as well as inapplicability of the sum-of-the-integrals objective function of the standard optimization formulation (Patriksson, 1994, pp. 52–53).
   That vehicle delays on links depend on the interaction of intersection flows is clear from the viewpoint of modern traffic science, and should be increasingly embarrassing to traffic network modelers. How to bring traffic network models into closer relationship with modern traffic science, as represented by the Highway Capacity Manual (TRB, 2000) and associated software (Akcelik and Associates, 2006; Dowling Associates, 2006) is a matter for investigation by specialists interested in bridging network equilibrium and traffic models.
4. Lastly, I come to the question of the representation of clock time in traffic equilibrium models. The original formulations of Beckmann and subsequent authors are now termed static models. These models represent a relatively long period of time in which flows are regarded as constant, such as the peak period or off-peak period. Generally, the period is at least one hour in length. In contrast, dynamic models seek to represent the interactions of traffic flows over much shorter time periods, such as one minute or five minutes. Static models are clearly a substantial and elegant simplification of reality, and are generally tractable. Solving dynamic models for large-scale networks remains a substantial challenge.

Dynamic traffic assignment (DTA) models have been the subject of ongoing research since the mid-1970s and intensive efforts since 1990. Nevertheless, DTA models have yet to achieve the level of acceptance in planning practice held by their static counterparts. Just how these models will progress and evolve in the future is a matter of very substantial speculation. Micro- and meso-simulation models are making contributions in this arena, offering alternatives to analytical formulations. Their discussion, however, lies well beyond what can be considered in this brief essay.

Many other areas of research and practice could be mentioned. The above four items are the focus of my current thinking. How these models evolve in the near and far term is a matter of chance and emphasis, as well as difficulty of the basic problems. Equally important is that we all be alert to significant advances in the state of the art, in order that the benefits of research findings can be put to good use in planning practice as early as possible.

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